# Currency Mismatches and Monetary Policy: A Tale of Two Equilibria*

Roberto Chang† — Rutgers University
Andrés Velasco‡ — Harvard University

This version: February 2005

**Abstract**

Recent papers have emphasized that the relative benefits of fixed versus flexible exchange rates depend on the currency denomination of assets and liabilities. At the same time, it is well understood that portfolio choices depend on anticipated exchange rate policy. What are the implications of the resulting interaction? We develop a simple model in which domestic agents choose how much to borrow in domestic or foreign currency. The central bank, in turn, chooses fixed or flexible exchange rates, taking the currency denomination of debts as given. We characterize the simultaneous determination of portfolios and exchange rate policy, and find that both floating exchange rates and fixed exchange rates can emerge as equilibrium outcomes. The analysis therefore shows how "fear of floating" may emerge endogenously and in association with a currency mismatch in assets and liabilities. We also find that, if equilibria with both fixed rates and floating rates coexist, the latter is Pareto superior. In that case, a precommitment to floating would be desirable to ensure that the "good" equilibrium obtains. Appropriate regulation of the currency denomination of debts would attain the same outcome.

*This paper was prepared for the San Francisco Federal Reserve Bank conference on Emerging Markets, June 2004. We are grateful to Guillermo Calvo, Enrique Mendoza and especially to Maurice Obstfeld, our discussant, for useful comments. Chang thanks Princeton University for hospitality and financial support while working on this project. We also thank the National Science Foundation and the Harvard Center for International Development for financial support.

†Email: chang@econ.rutgers.edu

‡Email: andres_velasco@harvard.edu

# 1 Introduction

Emerging market countries have trouble letting their exchange rates float, and many countries that claim to float do not deliver on that promise. That is the conclusion of much recent empirical work, starting with Stein et al. (1999) and Calvo and Reinhart (2002).[1] The reason, these papers argue, is a lethal mix of dollarization of liabilities and balance sheet effects: if corporate debts are denominated in dollars while the value of corporate assets depends on local currency (or if corporate revenues increase with the relative price of goods produced at home), sharp and unexpected currency movements matter for financial stability. The policy conclusion is that flexible exchange rates can be destabilizing, and therefore emerging market nations would be well advised to adopt alternative monetary arrangements, including currency boards and official dollarization.

Such a view has become extremely influential, but even its most ardent advocates understand that it is only half the story. The claim is that floating is not feasible *given* that debts are dollarized. But, presumably, borrowers choose the amounts of debt to issue as peso and dollar-denominated bonds taking into account the risk-return characteristics of these securities. Recognizing that variances and covariances (especially with consumption) should then matter, Ize and Levy-Yeyati (2003), Ize and Parrado (2003), and Morón and Castro 2003) have extended standard portfolio theory to model endogenous dollarization in emerging markets. Their approach, however, takes as given the structure of shocks and, more importantly for our discussion, monetary and exchange rate policies.

Since –as the recent debate has emphasized– exchange rate policies depend on portfolio choices but at the same time portfolio choices depend on anticipated exchange rate policies, the next question is inevitable: what are the implications of this interaction, if portfolios and exchange rate policy are both endogenous? In particular, what are the resulting policy outcomes? Is there a single outcome, or several ones? These are the issues on which this paper focuses.

We build an extremely simple model of a small open economy in which domestic residents borrow internationally by issuing bonds denominated in both home and foreign currency. The currency composition of the debt plays a nontrivial role because markets are incomplete: bonds are promises to nominal payoffs that can only imperfectly (or not at all) depend on the realization of the state of nature. We also assume sticky wages. So, as in textbook models, monetary and exchange rate policy matters: in the presence of external shocks flexible exchange rates stabilize labor supply and output at the expense of making exchange rates more volatile. But, as emphasized in the more recent literature, unexpected changes in exchange rates also affect wealth and exacerbate the volatility of domestic consumption if domestic residents are long in one currency and short in the other. Therefore, the optimal choice of exchange rate policy by a benevolent central bank depends on the existence and extent of

[1] See also Calvo (1999, 2000) and Krugman (1999, 2000).

currency mismatches. But in turn these are determined by the optimizing decisions of domestic borrowers and, hence, by their expectations about exchange rate policy.

The *equilibrium* outcome of this interaction is an exchange rate regime and a market allocation such that the market allocation is a competitive equilibrium given the exchange rate regime, and the central bank cannot increase social welfare by deviating to a different exchange regime. We assume that the central bank chooses the exchange rate regime (whether to fix the nominal exchange rate or to let it float and fix the domestic price level instead) after debts and wage contracts have been written. Then market expectations about exchange rate policy play a crucial role in shaping equilibria.

In the main version of our model, bonds are assumed to be non-contingent promises to either home or foreign currency. In that setting there is always an equilibrium with floating exchange rates. So, if agents expect the central bank will float, they arrange their wage and debt contracts accordingly, and given which the central bank indeed finds it optimal to float. But in some cases there is also an equilibrium with fixed exchange rates: if agents expect fixed rates, they may choose wages and portfolios that make it optimal for the central bank to fix *ex post*. That is, one can have multiple equilibrium policy regimes.

The intuition underlying equilibria with fixed exchange rates is particularly illuminating. If agents expect fixed exchange rates, home and foreign currency debts become perfect substitutes. This may result, in particular, in an asset-liability position that is very long in one currency in short in the other. But then an unanticipated switch from fixed to flexible rates raises the volatility of financial wealth and consumption. This effect can, if the currency mismatch in assets and liabilities is large enough, deter the central bank from disappointing expectations of a currency peg, which then become self fulfilling.

The equilibrium with fixed exchange rates, therefore, resembles “fear of floating.” As emphasized by Calvo and Reinhart (2002), such a fear may be associated with dollarized liabilities. But there are two significant differences between our analysis and previous discussions. First, in our model liability dollarization may not only give rise to fear of floating, but itself may emerge because of the rational anticipation of that fear. Second, in our model it is the existence of a currency mismatch in assets and liabilities (rather than liability dollarization per se) that generates fear of floating.

When there are multiple equilibria, they can be Pareto-ranked. We show that, if utility functions are quadratic or display constant relative risk aversion, the equilibrium involving flexible rates yields higher expected welfare. So if the economy is in a situation in which there are two equilibrium policy regimes, and arbitrary expectations cause the fixed rates equilibrium to materialize, then expected welfare will be inefficiently low. Welfare would increase if the central bank could pre-commit to float the currency, regardless of the composition of agents’ portfolios. Alternatively, appropriate regulation of the currency composition of debts can eliminate the Pareto inferior equilibrium with fixed exchange rates. In this sense, the model provides a novel justification for recent policy attempts at "de-dollarization."[2]

We also study the case in which domestic bonds are indexed to the price of home output. Equilibrium policy regimes turn out to be harder to pin down; still, under some further assumptions, we are able to identify conditions for fixed rates and flexible rates to be equilibrium policy regimes. Again, there is a range of parameters for which both regimes occur in equilibrium; in those cases, a policy of flexible rates again delivers higher welfare than do fixed rates.

The closest forerunner to our paper is by Corsetti and Pesenti (2002). They focus on the endogenous emergence of a currency area in a setting in which firms can choose to set prices in either domestic and foreign currency.[3] While our model and Corsetti and Pesenti's are rather different, there is a common intuition underlying the analysis of both: prior to the choice of an exchange rate regime, private agents make choices (the currency denomination of debt in our paper, the currency in which prices are fixed in Corsetti and Pesenti's) that determine their implicit claims to revenues denominated in different currencies. But once such claims are in place, they affect the relative benefits of flexible and fixed exchange rates. Accordingly, the analysis and results of Corsetti and Pesenti's model are similar to the results we find for our model, though the contexts are not the same.

Our discussion is close in spirit to that in Chamon and Hausmann (2005). In that paper, if domestic firms have large dollar liabilities, unexpected changes in the real exchange rate can drive the firms into costly bankruptcy. The central bank can react to shocks by allowing the interest rate or the exchange rate to move. If domestic firms expect a policy of stable exchange rates they will borrow in dollars, which ex post may cause the monetary authority to validate such expectations for fear of bankrupting the firms. Hence expectations of a particular policy can be self-confirming. This conclusion is similar to ours, but there are important differences between the two papers. Chamon and Hausmann focus on the decision by the central bank to let the exchange rate depreciate following a shock, while we focus on the choice of an exchange rate regime: fixing or floating. Therefore the expected volatility of the exchange rate (and of consumption) and the extend of currency mismatches (as opposed to dollarization *per se*) play a central role in our results, while they play no role in theirs. Finally, Chamon and Hausmann use an ad-hoc model with an ad-hoc policy objective function, while we develop a micro-founded model in which the objective function of the government is to maximize the expected utility of the representative agent in society.

Our policy message here is similar to that in Caballero and Krishnamurty

[2] Our model also provides a counter example to the conjecture, expressed by Tirole (2003) and others, that direct intervention in financial contracts is bound to be welfare-decreasing if such contracts are chosen by borrowers to maximize their own welfare.

[3] Corsetti and Pesenti find two equilibria. In one, firms preset prices in domestic currency only, and foreign-currency prices are determined by the law of one price. Then each central bank finds it optimal to let exchange rates float. In a second equilibrium firms preset prices in local currency; then a monetary union emerges as the optimal policy choice. Corsetti and Pesenti also find that the equilibrium with flexible rates is welfare superior to the equilibrium with fixed rates in their model.

(2004), which analyzes a model in which financial market imperfections lead agents to under-provide insurance against liquidity shocks. In that model floating the exchange rate is powerless to ameliorate shocks once the quantity of insurance has been chosen, but can help *ex ante* to induce agents to take greater precautions against shocks. Hence Caballero and Krishnamurty also argue for pre-committing to a float, though for reasons very different from ours.

The next section outlines the basic model, and section 3 presents the basic results. Section 4 extends the analysis to the case in which peso bonds are indexed, while section 5 concludes. Some technical material is delayed to an appendix.

# 2 The model

Consider a single-period, small open economy populated by households and firms. The representative household owns the typical firm and receives its profits.

There are two goods, one produced at home and one produced abroad. The two goods are both tradable and imperfect substitutes for each other. We assume that domestic households consume only the foreign good. This assumption, and others below, will allow us to derive closed form solutions to the model without affecting the main message of the paper.

There is a domestic currency, called *peso,* which is issued by a domestic central bank. There is also a foreign currency called *dollar.* Foreign goods have a constant price of one in terms of dollars, so we speak indistinctly of dollars and foreign goods.

To finance operations, at the beginning of the period under study home firms borrow from the world market, here represented by a continuum of risk-neutral lenders. The key assumption in this section is that the typical firm can borrow or lend in pesos or dollars. Therefore, the firm's optimal borrowing policy determines the degree of "dollarization" in the economy. This choice will be influenced by the firm's expectations about equilibrium prices and the exchange rate. These are determined by the monetary policy chosen by the central bank, which in turn takes into account the degree of dollarization.

## 2.1 Firms

The representative firm has access to the technology

$$Y = AK^{\alpha}L^{1-\alpha} \tag{1}$$

where $0 < \alpha < 1$ and $A$ is a positive parameter. For simplicity, we also assume that the capital stock $K$ is of fixed size. Households are heterogeneous in the labor services they provide, and the input $L$ is an aggregate of the services of the different households in the economy:

$$L = \left[\int_0^1 L_i^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

where we have indexed workers by $i$ in the unit interval, $L_i$ denotes the services purchased from household $i$, and $\theta > 1$ is the elasticity of the demand for household $i$'s services.

Let $W_i$ denote the wage charged by worker $i$ and $W$ denote the aggregate wage, that is, the minimum cost of a unit of the $L$ aggregate, expressed in terms of pesos. Cost minimization yields the demand for household $i'$s labor:

$$L_i = \left(\frac{W_i}{W}\right)^{-\theta} L \tag{2}$$

The firm has no capital to start with, and all capital is imported. So the firm must finance capital purchases by borrowing abroad. To do this, at the beginning of the period, the firm sells bonds denominated either in pesos ($B$) or dollars ($B^*$). A peso (resp. dollar) bond is a promise to a peso (resp. dollar) at the end of the period. Note that, in assuming that bond payments cannot be arbitrary functions of the state of nature, we are imposing market incompleteness, which implies that currency composition plays a nontrivial role.

We assume that the world interest rate in dollars is zero, so a dollar bond must sell for one dollar. Letting $Q$ denote the (beginning-of-period) price, in dollars, of a peso bond, it follows that

$$QB + B^* = K, \tag{3}$$

End-of-period firm profits, also in dollars, are denoted by $\Pi$ and given by

$$S\Pi = PY - WL - SB^* - B, \tag{4}$$

where $S$ is the exchange rate (in pesos per dollar) and $P$ the peso price of home output. As usual, we assume that $L$ is chosen at the end of the period, after uncertainty has been realized.

Since the firm is owned by the representative household, it is natural to assume its objective function is $E\{u'(C)\Pi\}$, where $u'(C)$ represents the marginal utility of the household's consumption, to be derived below, and $E\{.\}$ denotes the expectation at the beginning of the period. Hence the firm chooses $B$, $B^*$, and a contingent plan for $L$ to maximize $E\{u'(C)\Pi\}$ subject to 2, 3, and 4. The solution is

$$E\left\{u'(C)\left(Q - \frac{1}{S}\right)\right\} = 0, \tag{5}$$

and

$$WL = (1-\alpha) PY, \tag{6}$$

which are standard. In particular, 5 characterizes the firm's optimal borrowing policy: an additional unit of peso bonds raises $Q$ dollars at the beginning of the period, and requires a repayment of $1/S$ dollars at the end of the period. At the margin, the expected net gain from such issue must be zero.

## 2.2 Households

As already mentioned, households provide differentiated labor services, so each household enjoys some monopoly power in the labor market. Assume that, at the beginning of the period, each household sets a wage in pesos and commits to satisfy demand forthcoming at that wage at the end of the period. The household then consumes the dollar value of its labor income plus firm profits.

Formally, household $i$ chooses its wage, $W_i$, to maximize

$$E\left\{u(C_i) - \left(\frac{\theta-1}{\theta}\right) v\left(L_i\right)\right\}$$

subject to

$$SC_i = S\Pi + W_i L_i \tag{7}$$

and to the labor demand function 2. The functions $u$ and $v$ satisfy usual assumptions, and $\theta > 1$.

The optimal wage solves

$$WE\left\{\frac{u'(C)L}{S}\right\} = E\left\{Lv'(L)\right\}, \tag{8}$$

where we have imposed symmetry and eliminated $i$ subscripts.

## 2.3 Foreign lenders

Foreign lenders are risk neutral and only care about foreign goods. Hence they will buy peso bonds if and only if their expected return, in dollars, equals the dollar world return of zero. Therefore,

$$Q = E\left\{\frac{1}{S}\right\}. \tag{9}$$

## 2.4 Market clearing

Since local residents do not consume home goods, the demand for home output comes from foreigners. We assume that the dollar value of the foreign demand for home output is exogenous and given by a random variable $X$, with $E\{X\} > K$. Note that $X$ is the only source of uncertainty in the model.

The market for home goods clears if

$$PY = SX. \tag{10}$$

Competitive equilibrium is well defined once monetary policy is specified. Before proceeding to the analysis of policy, note that in any competitive equilibrium household consumption is obtained by combining 3, 4, 7, and 9

$$C = \left(\frac{P}{S}\right) Y - \left(\frac{B}{S} + B^*\right) = X - K + \left(E\left\{\frac{1}{S}\right\} - \frac{1}{S}\right) B \tag{11}$$

The first equality says that consumption equals the dollar value of output minus the cost of servicing the foreign debt. But 10 implies that, again in equilibrium, the dollar value of output is given by $X$, while the debt burden is equal to the initial cost of investment ($K$) minus the capital gains or losses on peso debt associated with an exchange rate surprise (the last term on the RHS).

Furthermore, since foreign lenders are risk neutral and rational there cannot be expected capital gains or losses on peso debt. Accordingly, in any competitive equilibrium,

$$E\{C\} = E\{X\} - K$$

So in this model currency denomination of debt cannot affect the expected value of consumption. But it can affect the variability of consumption, in a manner that will depends on the distribution of the exchange rate and, hence, on policy.

# 3 Equilibrium monetary policy

We restrict attention to two alternative policies: a *fixed exchange rate*, defined as a policy that keeps $S$ constant, and a *flexible exchange rate*, which keeps $P$ constant.[4] This section studies the equilibrium outcomes under either flexible rates or fixed rates. Then we ask whether either alternative is an *equilibrium* policy.

## 3.1 Competitive equilibrium with flexible exchange rates

Flexible exchange rates are defined as a regime in which the price of home output is constant. The resulting outcomes will be identified by tildes. Clearly the price level is immaterial, so we normalize $\tilde{P} = 1$.

Equilibrium in the labor market (equation 6) reduces to

$$\tilde{W}\tilde{L} = (1-\alpha)\tilde{Y}. \tag{12}$$

But by 1 output is a function only of $\tilde{L}$, and $\tilde{W}$ is set in advance. It follows that both $\tilde{Y}$ and $\tilde{L}$ must be constant under flexible rates.[5] Intuitively, since monetary policy stabilizes the price of home output and nominal wages are fixed in pesos, the real (product) wage is constant. Hence the marginal product of labor and (since there are no productivity shocks) labor demand and output must be constant too.

The nominal exchange rate, however, is not constant. By 10,

$$\tilde{S} = \frac{\tilde{Y}}{X}, \tag{13}$$

[4] As discussed by Woodford (2004), for policies in this class there is no need to specify the sources of the demand for domestic currency, which would only identify the supply of currency needed to implement those policies.

[5] Given $\tilde{W}$, $\tilde{Y}$ and $\tilde{L}$ are the solution of 1 and 12.

so the distribution of the exchange rate is given by the distribution of exports $X$. In particular, the nominal exchange rate appreciates ($S$ falls) when exports are higher.

To find the currency composition of debt, use 9 in the first-order condition 5:

$$E\left\{u'\left(\tilde{C}\right)\left[E\left\{\frac{1}{\tilde{S}}\right\}-\frac{1}{\tilde{S}}\right]\right\}=0.$$

This implies

$$Cov\left\{\frac{1}{\tilde{S}},\,u'(\tilde{C})\right\}=0$$

Intuitively, equilibrium portfolios are such that the marginal utility of consumption is orthogonal to unexpected movements in the exchange rate. It turns out that the firm accomplishes this by choosing $B$ and $B^*$ so as to make consumption constant. From 11 and the fact that under flexible exchange rates $\tilde{P}=1$, consumption is constant if

$$\tilde{B}=\tilde{Y},$$

which is therefore the optimal portfolio allocation.

The intuition is straightforward. Given that wages are fixed and labor effort is constant, the only risk the household faces is exchange rate risk, which can cause the price of domestic output in terms of consumption goods to fluctuate. The firm eliminates this risk on behalf of its owner, the household, by borrowing an amount in pesos equal to the (constant) value of output. This way the household is fully hedged against real and nominal exchange rate risk.

The corresponding constant consumption level is, by 11 and 13,

$$\tilde{C}=E\{\tilde{C}\}=E\left\{X\right\}-K.$$

So with flexible rates the household consumes the expected dollar value of home output minus the cost of capital.

Note that under this allocation $\tilde{Q}\tilde{B}=\tilde{Y}E\left\{1/\tilde{S}\right\}=E\left\{X\right\}>K$ is necessary for consumption to be positive. This means that initially the firm sells peso bonds with a higher value than its total foreign liability $K$, and devotes some of the proceeds to buying dollar assets. So the firm becomes a net creditor in dollars and a net debtor in pesos. This ensures that it enjoys capital gains when the exchange rate depreciates, which is precisely the time when the dollar value of national income is low.

To complete the characterization of the economy under flexible exchange rate, observe that nominal wages are given by the optimal wage setting condition 8, which reduces to:[6]

$$\tilde{W}=\frac{v'\left(\tilde{L}\right)}{u'\left(\tilde{C}\right)E\left\{1/\tilde{S}\right\}}=\frac{v'\left(\tilde{L}\right)\tilde{Y}}{u'\left(\tilde{C}\right)E\left\{X\right\}}. \tag{14}$$

[6] Note that this is not a closed form solution since $\tilde{L}$ and $\tilde{Y}$ depend on $\tilde{W}$. To obtain $\tilde{L}$, $\tilde{Y}$ and $\tilde{W}$ , 1 , 12 and 14 must be solved simultaneously.

## 3.2 Flexible exchange rates and equilibrium policy

Suppose that for some reason agents expect a policy of floating and set wages and portfolios accordingly. Given those expectations, will the monetary authority deliver the expected policy *ex-post*? In other words, are expectations of floating self-fulfilling? To answer these questions, here we consider whether floating is an equilibrium policy.

Doing so requires that we be more precise about the timing of actions. We assume the following timing: the period under analysis starts with a contracting stage in which firms and foreign lenders agree on debt contracts and workers set the value of their nominal wages. Then the central bank chooses the policy regime: either fixed or flexible exchange rates. The authorities take no other action after that. Finally, uncertainty about exports is realized and production, trade and consumption take place. Figure 1 depicts the time line.

FIGURE 1

Agents write contracts —> Central Bank sets policy regime —>Uncertainty is realized —>Trade, production and consumption take place.

Some elaboration is warranted. Our assumption that the central bank chooses a policy regime before observing the realization of exogenous shocks is intended to capture the idea that monetary policy is often formulated in terms of relatively simple rules (here, either to fix $P$ or to fix $S$). As emphasized in the recent literature (e.g. Woodford 2004), this can be justified in several ways, such as the impossibility of making policy depend on all relevant shocks to the economy. In turn, our assumption that the policy regime is chosen after wage and debt contracts are written reflects the fact that, in reality, it is usually the case that at least some wage and debt contracts are already in place whenever a central bank has the opportunity to change the policy rule.[7]

In choosing the policy regime the central bank maximizes the expected welfare of its representative citizen. Floating exchange rates are then an *equilibrium policy* if, given that portfolios and wage contracts were optimally written in the expectation of flexible rates, the central bank has the incentive to deliver precisely that policy, instead of deviating and applying fixed exchange rates.[8]

We also require that if the central bank is to deviate from flexible to fixed rates or vice versa, the deviation must leave the expected dollar value of pesos unchanged at its pre-deviation level. In other words, it must involve no expected

[7] The essential aspect of our timing assumptions is, therefore, that *some* wage and debt contracts are renegotiated less often than monetary policy can be reformulated. This would be the case, for instance, if there are some wage and debt contracts that last for a year, while the central bank meets every week. This is clearly realistic and also consistent with the "fear of floating" literature, which is predicated on the concern that exchange rate policy has to be adopted once large dollar debts are in place.

[8] This definition of equilibrium policy is the same as in Chari and Kehoe (1990) and Stokey (1991).

expropriation of foreign lenders. One justification for this restriction is that it may be very costly for the country to default, implicitly or explicitly, on its foreign debt obligations. But we also adopt the restriction to impose discipline in our analysis of possible deviations away from fixed and flexible exchange rates. If the central bank deviates from fix to flex, it has to choose the level at which it will set the price level $P$; if it deviates from flex to fix, it has to choose the level at which it will set the nominal exchange rate $S$. These new settings of $P$ and $S$ are free parameters (in principle the central bank could choose anything), but different settings can make deviating more or less attractive –for instance, by expropriating foreigners more or less. To ensure that we are comparing apples with apples we require that the settings of $P$ and $S$ be such that no expropriation of foreigners is involved.[9] This rule also helps us focus on domestic policy concerns, abstracting from (and providing a complementary perspective to) the better known time inconsistency issues associated with foreign debt.

Denote outcomes under a deviation to fixed exchange rates by an overbar. By the no-expropriation assumption, after a policy deviation $1/\bar{S}$ must equal $E\left\{1/\tilde{S}\right\}$ which, given 13, requires that the exchange rate be fixed at

$$\bar{S} = \frac{\tilde{Y}}{E(X)}. \tag{15}$$

Expressions 6 and 10 must also hold, so labor effort is given by

$$\bar{L} = \left(\frac{1-\alpha}{\tilde{W}}\right)\bar{S}X, \tag{16}$$

where the nominal wage rate is that associated with flexible rates, since it was set before the deviation. Using the definition of $\bar{S}$ from 15 in 16 we then obtain labor effort under a deviation:

$$\bar{L} = \left(\frac{1-\alpha}{\tilde{W}}\right)\left(\frac{X}{E\{X\}}\right)\tilde{Y}.$$

Hence labor effort becomes a linear function of $X$, with expectation

$$E\left\{\bar{L}\right\} = \left(\frac{1-\alpha}{\tilde{W}}\right)\tilde{Y} = \tilde{L}.$$

In words, the deviation to fixed exchange rates keeps expected labor supply the same, but increases the variability of labor effort. The latter occurs because fixing the exchange rate means that the price of home output, and hence the real wage, must fluctuate in order to accommodate shocks to export demand.

[9] What is important here is the consistency implied by the rule, not the fact that it allows for exactly *zero* expropriation. Any rule that permitted some fixed (positive or negative) amount of expropriation would serve the same disciplining purpose. But assuming zero expropriation seems natural, and can be justified by appealing to costs of international default.

Using 11 and the fact that the deviation keeps $1/\bar{S}$ at $E\left\{1/\tilde{S}\right\}$, consumption after a deviation to fixed rates is given by

$$\bar{C} = X - K$$

Taking expectations of this expression we again have that

$$E\left\{\bar{C}\right\} = E\left\{X\right\} - K = \tilde{C}$$

Hence, the deviation also causes a mean-preserving spread in consumption.

In short: by deviating from flexible rates to fixed rates when households had expected the former, the monetary authority induces volatility into labor supply and consumption without changing the expected value of either variable. Since volatility decreases expected utility, the policymaker can only decrease expected utility by switching to fixed rates. It follows that flexible exchange rates are *always* an equilibrium policy regime.

## 3.3 Competitive equilibrium under a fixed exchange rate

Now consider a policy of fixing the exchange rate at $S = \bar{S} = 1$ (we abuse notation and again use overbars to denote fixed rates). Then nominal demand reduces to

$$\bar{P}\bar{Y} = \bar{S}X = X, \tag{17}$$

and 6 gives labor effort:

$$\bar{L} = \frac{(1-\alpha)X}{\bar{W}}. \tag{18}$$

So labor effort becomes proportional to the demand for home output. Employment must fluctuate since, with fixed exchange rates, the price of home output and the product wage must accommodate changes in demand.

Expression 11 for consumption becomes

$$\bar{C} = X - K. \tag{19}$$

Home consumption is equal to $X$ minus the constant value of investment. The reason is clear from 11: since the exchange rate is fixed, home agents experience no unanticipated capital gains nor losses.

Now $B$ and $B^*$ are *indeterminate*, since bonds in pesos and dollars are perfect substitutes. This means that in practice portfolio composition may be pinned down by considerations outside the model.

Finally, nominal wages are given by 8, which reduces to

$$\bar{W} = \frac{E\left\{\bar{L}v'(\bar{L})\right\}}{E\left\{u'(\bar{C})\bar{L}\right\}} = \frac{E\left\{Xv'(\bar{L})\right\}}{E\left\{u'(\bar{C})X\right\}}. \tag{20}$$

### 3.4 Is a fixed exchange rate an equilibrium policy?

To check whether a fixed rate is an equilibrium policy, consider a deviation to a flexible exchange rate, assuming that in such a deviation $E\left\{1/\tilde{S}\right\} = 1/\bar{S} = 1$. Again, the justification for the restriction is to ensure that the deviation imposes no expected expropriation on foreigners.[10]

After a switch, 10 must hold, so that

$$\frac{1}{\tilde{S}} = \frac{X}{\tilde{P}\tilde{Y}}.$$

By assumption, the expectation of the LHS after a deviation must equal unity. But the deviation also implies that both $\tilde{P}$ and $\tilde{Y}$ are constant. Hence, taking expectations on both sides of the preceding equation, and using 10 again, we find the exchange rate associated with the deviation:

$$\tilde{S} = \frac{E\{X\}}{X}. \tag{21}$$

Applying this to 6 we have that labor supply is given by

$$\tilde{L} = \frac{(1-\alpha)\tilde{S}X}{\bar{W}} = \frac{(1-\alpha)E\{X\}}{\bar{W}}.$$

Comparing this last expression with 18 we see that after a deviation to floating labor effort is no longer variable, and its mean value does not change. Hence there is a "temptation" to abandon fixed rates.

To find the effect on consumption of a switch to flexible rates, use 21 in 11 evaluated at $E\left\{1/\tilde{S}\right\} = 1$. This yields

$$\tilde{C} = X\left(1 - \frac{\bar{B}}{E\{X\}}\right) + \bar{B} - K. \tag{22}$$

Hence the effect of the deviation on consumption depends on the degree of dollarization, which is indeterminate under fixed rates. But notice that $E\left\{\tilde{C}\right\} = E\{X\} - K$, so the deviation keeps the *expected value* of consumption constant. Therefore, the expected utility from consumption may increase or fall depending on the response of the *variability* of consumption to the deviation.

In turn, the response of consumption variability depends on the currency composition of the debt and, in particular, the extent of currency mismatches: from 22 we see that, after a deviation, the variance of consumption falls if $0 < B < 2E\{X\}$, and it increases otherwise. If the variance falls, the policymaker will unambiguously want to deviate, since that would reduce the variance of both consumption and labor supply while preserving their expected values. For instance, if (by fluke) agents had adopted the portfolio that corresponds to the

[10] And now, in a further abuse of notation, we use tildes to identify the outcomes of a deviation to flexible rates.

expectation of flexible rates, the variance of consumption after the switch would fall to zero, and deviating from fixed rates would be optimal for the policymaker.

It follows that a necessary condition for fixed rates to be an equilibrium policy is either $B < 0$ or $B > 2E\{X\}$. Under further assumptions discussed below, $B < 0$ or $B > 2E\{X\}$ is also sufficient. The intuition for this condition is that, in this setting, the only deterrent that can keep the central bank from deviating is that floating exchange rates may result in very variable capital gains or losses, therefore raising the variability of consumption. But this effect obtains only if debts exhibit a sufficiently large currency mismatch. Note that the currency mismatch makes no difference to borrowers or lenders in equilibrium, but it deters the government from abandoning fixed exchange rates.

The case in which $B < 0$ is perhaps the more interesting one: the representative agent has gross assets in pesos and gross debts in dollars, a case that Calvo (2001) termed *liability dollarization*. In this sense, our analysis shows how a sufficiently strong degree of liability dollarization may be associated to *fear of floating* as Calvo and Reinhart (2002) have emphasized. But a difference between our results and those of previous studies is that, in our model, liability dollarization not only gives rise to fear of floating, but also emerges because of the correct expectation of that fear.

Our discussion is also novel in that it emphasizes the impact of the choice of the exchange rate regime on second, not first, moments. Previous discussions have assumed not only that there is liability dollarization but also that letting the exchange rate float is expected to result in a depreciation of the currency, so that a switch from fixed to flexible rates causes an expected wealth loss for domestic borrowers. Here, by contrast, a policy switch has (by assumption) no effect on the expected value of the exchange rate, and therefore on expected real allocations . The switch has effects on expected welfare, however, because it affects the higher moments of the exchange rate and of real variables such as consumption, and because households are risk averse.

Hence it is the *currency mismatch* of debt, rather than liability dollarization itself, that generates fear of floating in our model. Since a switch from fixed to flexible rates stabilizes labor effort and has effects only on the higher moments of the exchange rate distribution, it can only reduce welfare through the higher moments of the distribution of capital gains and losses, which depend on the currency mismatch. While liability dollarization implies a currency mismatch, the latter can also emerge in the opposite case of large dollar assets and peso debts; this is why fixed exchange rates also emerge as an equilibrium policy if $B > 2E\{X\}$.

Summarizing: if fixed exchange rates induce a sufficiently large mismatch in the currency denomination of debt (either $B < 0$ or $B > 2E\{X\}$), a switch from fixed to flexible rates induces a mean-preserving spread in consumption. Since the deviation keeps labor effort at its mean value under fixed rates, fixed exchange rates may or may not be an equilibrium. This depends on the parameters of the model and, in particular, on the utility cost associated with consumption fluctuations relative to labor effort fluctuations (determined by the shape and curvature of $u$ and $v$). But also, and importantly for our purposes, it depends on the currency composition of the firm's debts, which is not uniquely pinned down in equilibrium.

If a fixed exchange rate is in fact an equilibrium, then there are multiple equilibria in policy regimes, since flexible exchange rates are always an equilibrium. In such a case, animal spirits play a role: if agents expect fixed rates (and the denomination of the debt exhibits a sufficiently large currency mismatch) the government will indeed deliver fixed rates; if agents expect flexible rates, the government will choose flexible rates.

*Example:* Suppose

$$u(C) = \frac{C^{1-\rho}}{1-\rho}$$

and

$$v(L) = \frac{\kappa}{2} L^2.$$

The equilibrium wage is then given by inserting 19 and 18 in 20:

$$\bar{W}^2 = \frac{\kappa(1-\alpha)E\left\{X^2\right\}}{E\left\{X\right\}(X-K)^{-\rho}}.$$

One can then calculate that switching from fixed rates to fixed rates increases the expected cost of labor effort by

$$E\left\{v(\bar{L})\right\} - E\left\{v(\tilde{L})\right\} = \frac{(1-\alpha)}{2}\left(E(X)(X-K)^{-\rho}\right)\frac{Var\left(X\right)}{E\left\{X^2\right\}} > 0.$$

The switch also causes an expected change in the utility of consumption of

$$E\left\{u(\bar{C})\right\} - E\left\{u(\tilde{C})\right\} = \frac{1}{1-\rho}\left[E\left\{(X-K)^{1-\rho}\right\} - E\left\{X - K + B\left(1 - \frac{X}{E(X)}\right)\right\}\right]^{1-\rho}$$

The arguments above imply that $E\left\{u(\bar{C})\right\} - E\left\{u(\tilde{C})\right\} > 0$ if $B < 0$ or $B > 2E\left\{X\right\}$. Assuming that either condition holds, fixed exchange rates are an equilibrium policy if

$$E\left\{u(\bar{C})\right\} - E\left\{u(\tilde{C})\right\} - \left(\frac{\theta-1}{\theta}\right)\left[E\left\{v(\bar{L})\right\} - E\left\{v(\tilde{L})\right\}\right] > 0.$$

For any given $\bar{B}$, this condition is satisfied if either $\theta$ or $\alpha$ are close enough to one.

### 3.5 Welfare

Our analysis implies that both flexible rates and fixed exchange rates can be equilibrium policies in our model. Importantly, they can also be Pareto ranked when they coexist.

We have already shown that expected consumption must be the same under both policy regimes, and that flexible exchange rates completely stabilize both consumption and labor effort. But it is hard to compare the mean value of labor effort under the two regimes. Hence the welfare ranking may depend on functional forms and parameter values.

However, there is no ambiguity if utility functions are quadratic or display constant relative risk aversion. In those cases flexible rates perform better. To see this formally, assume first that preferences are of the form

$$E\left\{\frac{C^{1-\rho}}{1-\rho}-\left(\frac{\theta-1}{\theta}\right)\frac{L^{1+\chi}}{1+\chi}\right\},\ \ \rho>0,\,\chi>0. \tag{23}$$

Under floating, 14 and 12 yield

$$\tilde{L}^{1+\chi}=(1-\alpha)u'(E\left\{X\right\}-K)E\left\{X\right\}, \tag{24}$$

where we have used the fact that with the assumed utility function $Lv'(L)=L^{1+\chi}$, and also the fact that under floating $\tilde{C}=E\left\{X\right\}-K$. Using 24 in 23 yields

$$E\left\{U^{\text{flex}}\right\}=\psi\frac{\left(E\left\{X\right\}-K\right)^{1-\rho}}{1-\rho}-(1-\psi)\frac{K\left(E\left\{X\right\}-K\right)^{-\rho}}{1-\rho}, \tag{25}$$

where $\psi\equiv1-(1-\rho)\left(\frac{1-\alpha}{1+\chi}\right)\left(\frac{\theta-1}{\theta}\right)>0$.

With analogous steps one can derive an expression for expected utility under fixing, which is

$$E\left\{U^{\text{fix}}\right\}=\psi\frac{E\left\{\left(X-K\right)^{1-\rho}\right\}}{1-\rho}-(1-\psi)\frac{KE\left\{\left(X-K\right)^{-\rho}\right\}}{1-\rho}. \tag{26}$$

Comparing 25 and 26 we see that $EU^{\text{flex}}>EU^{\text{fix}}$. The appendix analyzes the case in which both $u\left(C\right)$ and $v\left(L\right)$ are quadratic, and shows that $E\left\{U^{\text{flex}}\right\}>E\left\{U^{\text{fix}}\right\}$ also. We conclude that for two broadly used classes of preferences, the regime with flexible exchange rates yields higher expected welfare.

If both flexible and fixed rates are equilibrium policy regimes, benevolent policymakers must endeavor to convince agents that floating will indeed be the chosen policy. One alternative is to commit to a flexible regime. If this is not possible, direct regulation of portfolios so as to make flexible rates optimal *ex post* could also be desirable. This suggests that some recent attempts at "dedollarizing" financial contracts may be justified as devices to avoid coordination failures.

# 4 Indexed bonds

Readers might wonder whether the results on the multiplicity of equilibrium policy regimes are an artifice of the indeterminacy of portfolios under fixed rates. This is not so, as we show in this section by extending the model to a menu of assets that ensures that portfolios are always fully determined. We replace peso bonds with bonds that have payoffs indexed to the price of the domestic good. Such bonds are often present in emerging markets, so this variant of the model is not only of theoretical interest but also relevant in practice.

To be precise, we now assume that the representative firm sells dollar bonds and indexed bonds. An indexed bond is a promise to $P$ pesos at the end of the period. Rather than developing the model from scratch, we simply write down the equilibrium conditions that differ from those of the earlier formulation. Foreign lenders again arbitrage the returns on both kinds of loans, so the initial price of an indexed bond, in dollars, must equal the expected terms of trade $E\{P/S\}$.

The optimal wage setting condition 8 remains the same, while the optimal portfolio condition 5 becomes

$$E\left\{u'(C)\left(E\left\{\frac{P}{S}\right\}-\frac{P}{S}\right)\right\}=0, \tag{27}$$

Market clearing is still given by 10, while expression 11 for consumption becomes

$$C=X-K+B\left(E\left\{\frac{P}{S}\right\}-\frac{P}{S}\right) \tag{28}$$

where $B$ now denotes the outstanding number of indexed bonds.

## 4.1 A flexible exchange rate once again

Consider first a flexible rates regime with $\tilde{P}=1$. Assuming that this policy is credible and indeed carried out, indexed bonds become identical to peso bonds. Hence the outcomes are just the same as with flexible exchange rates in the model with peso bonds, characterized in subsection 3.1. Notice in particular that $\tilde{B}=\tilde{Y}$ –that is, indexed bonds in the portfolio are equal to the value of home output.

Indexed bonds do make a difference, however, in analyzing whether flexible rates are an equilibrium policy regime. Consider the implications of a deviation towards fixed exchange rates. Again to prevent expected expropriation of foreign lenders, we assume that such a deviation leaves the expected terms of trade, $P/S$, unchanged. This requires, using overbars once more to denote the consequences of a deviation to fixed rates,

$$E\left\{\frac{\bar{P}}{\bar{S}}\right\}=E\left\{\frac{1}{\tilde{S}}\right\}=\frac{E\{X\}}{\tilde{Y}}, \tag{29}$$

where the last equality follows from 10.

To solve for the consequences of a deviation, note that 6 implies

$$\bar{L}=\frac{(1-\alpha)\bar{S}X}{\tilde{W}}. \tag{30}$$

Moreover, from 10, we have

$$\bar{P} = \frac{\bar{S}X}{\bar{Y}} = \frac{\bar{S}X}{AK^{\alpha}\bar{L}^{1-\alpha}} = \left(\frac{\bar{S}X}{\tilde{Y}}\right)^{\alpha}, \tag{31}$$

where the last equality follows from 30, 1 and 12. It follows that

$$\frac{\bar{P}}{\bar{S}} = \left(\frac{X}{\tilde{Y}}\right)^{\alpha} \bar{S}^{\alpha-1}$$

Taking expectations and using 29 one obtains the nominal exchange rate under a feasible deviation:

$$\bar{S} = \tilde{Y}\left(\frac{E\{X^{\alpha}\}}{E\{X\}}\right)^{\frac{1}{1-\alpha}}. \tag{32}$$

Inserting this value in 31 and simplifying one gets the price level after a deviation:

$$\bar{P} = X^{\alpha}\left(\frac{E\{X^{\alpha}\}}{E\{X\}}\right)^{\frac{\alpha}{1-\alpha}}.$$

So, in particular,

$$E\{\bar{P}\} = \left(\frac{E\{X^{\alpha}\}}{[E\{X\}]^{\alpha}}\right)^{\frac{1}{1-\alpha}} < 1. \tag{33}$$

Hence a switch from flexible rates to fixed rates implies a fall in the expected price of home output. It follows that the expected real wage rises, and expected labor effort falls. Formally, from 30, 32 and the definition of $\tilde{Y}$ one can derive

$$\bar{L} = \tilde{L}\frac{X}{E\{X\}}E\{\bar{P}\}. \tag{34}$$

Taking expectations we have

$$E\{\bar{L}\} = \tilde{L}E\{\bar{P}\} < \tilde{L}.$$

Hence, the deviation to fixed rates implies that labor effort becomes variable but, in contrast with the case of peso bonds, the mean value of labor effort falls. The reduction in mean labor effort is welfare-improving, making a switch towards fixed rates attractive.[11]

As in the case of peso bonds, the switch from flexible to fixed exchange rates causes a mean-preserving spread in consumption (the proof is similar to the one in the case of peso bonds and left to the interested reader.) Additional consumption variability makes expected welfare fall and reduces the desirability of the deviation, as in the case of peso bonds. However, with indexed bonds,

[11] The fact that an increase in labor supply is welfare-decreasing might seem surprising, since the model features imperfect competition in the labor market, which causes equilibrium labor supply to be too low relative to the planner's solution. But in this model the dollar value of domestic production is given by 10. Hence working more just causes the terms of trade to turn against the country, without any benefit for consumption.

mean labor effort falls. Flexible rates are an equilibrium if the utility benefit associated with the smaller labor effort is less than the cost associated with increased variability in both consumption and labor.

As a special case, assume that

$$v(L) = \phi L^{1+\chi} \tag{35}$$

where $\phi > 0$ is an arbitrary constant and $\chi > 0$. Assume also that $X$ is lognormally distributed. Then, as we show in the appendix, the sign of $E\left\{v(\bar{L})\right\} - v(\tilde{L})$ equals the sign of $\chi - \alpha$. A switch from flexible rates to fixed rates increases or leaves the same the expected cost of effort if $\chi \geq \alpha$. Since the switch always causes a mean-preserving spread in consumption, then $\chi \geq \alpha$ is sufficient for the switch to be welfare-decreasing –that is, for flexible rates to be an equilibrium policy.

The intuition is that the larger is $\chi$, the larger is the utility cost of the increased variability of labor effort under a deviation. On the other hand, by 30, a larger $\alpha$ results in smaller fluctuations in labor effort in a deviation. So the cost of a switch from a flexible to a fixed exchange rate increases with $\chi$ and falls with $\alpha$.

## 4.2 A fixed exchange rate once again

Consider next the policy of fixing the exchange rate at $\bar{S} = 1$. Condition 18 still gives labor effort, and 28 implies that

$$\bar{C} = X - K + \bar{B}\left(E\left\{\bar{P}\right\} - \bar{P}\right). \tag{36}$$

This expression shows that, in contrast with the case of nominal peso bonds, the currency composition of the debt matters here even with fixed exchange rates. The central bank can peg the nominal exchange rate but not the terms of trade; the latter can affect capital gains or losses if peso bonds are indexed.

As a consequence, $\bar{B}$ is not indeterminate. Instead, it must be set to satisfy the condition 27, which here reduces to

$$Cov\left(u'(\bar{C}), \bar{P}\right) = 0. \tag{37}$$

The price of home output follows from 17 and the production function:

$$\bar{P} = \frac{X}{\bar{Y}} = \frac{X^{\alpha}}{AK^{\alpha}}\left(\frac{\bar{W}}{1-\alpha}\right)^{1-\alpha}. \tag{38}$$

The rest of the analysis turns out to be more difficult than before, so we assume from now on that $u(C)$ is quadratic (at least in the relevant range). Then $u'$ is linear in $C$, and expression 37 reduces to

$$Cov\left(\bar{C}, \bar{P}\right) = 0.$$

That is, equilibrium portfolios must be set so that consumption is orthogonal to the terms of trade. Using the previous expression for $\bar{C}$ one readily finds that the stock of indexed bonds in the equilibrium portfolio is

$$\bar{B} = \frac{Cov(X, \bar{P})}{Var\left(\bar{P}\right)}$$

This is intuitive: with quadratic utility, $\bar{B}$ must be chosen to minimize the variance of consumption which, from 36, is the variance of $X - \bar{B}\bar{P}$. Hence $\bar{B}$ is the coefficient of a linear regression of $X$ on $\bar{P}$.

Given 38, the preceding expression can be written as

$$\bar{B} = \frac{Cov(X, X^{\alpha})}{Var\left(X^{\alpha}\right)} AK^{\alpha} \left(\frac{1-\alpha}{\bar{W}}\right)^{1-\alpha}. \tag{39}$$

Replacing 39 in 36 yields equilibrium consumption:

$$\bar{C} = X - K + \frac{Cov(X, X^{\alpha})}{Var\left(X^{\alpha}\right)} \left[E\left\{X^{\alpha}\right\} - X^{\alpha}\right] \tag{40}$$

Now consider a deviation to flexible rates, imposing once more the restriction of no expropriation of foreign lenders, which requires that the post-deviation expected value $E\left\{\tilde{P}/\tilde{S}\right\}$ must equal $E\left\{\bar{P}\right\}$.

After the deviation, 6 must hold, which together with the production function yields labor effort:

$$\tilde{L} = A^{\frac{1}{\alpha}} K \left(\frac{1-\alpha}{\bar{W}}\right) \tilde{P}^{\frac{1}{\alpha}}, \tag{41}$$

where $\tilde{P}$ is the price level after the deviation, to be determined shortly.

Since 10 must hold, $\tilde{P}/\tilde{S} = X/\tilde{Y}$. Taking expectations on both sides and using the production function and 41 one obtains

$$E\left\{\frac{\tilde{P}}{\tilde{S}}\right\} = \frac{E\left\{X\right\}}{KA^{\frac{1}{\alpha}}} \left(\frac{(1-\alpha)\tilde{P}}{\bar{W}}\right)^{-\frac{1-\alpha}{\alpha}}.$$

But this has to be equal to $E\left\{\bar{P}\right\}$, where $\bar{P}$ is given by 38. So, taking expectations in 38, equating the result to the preceding equation and rearranging gives the required value of $\tilde{P}$:

$$\tilde{P} = \left(\frac{E\left\{X\right\}}{E\left\{X^{\alpha}\right\}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{AK^{\alpha}} \left(\frac{\bar{W}}{1-\alpha}\right)^{1-\alpha}.$$

Replacing in the equation for $\tilde{L}$ above we obtain

$$\tilde{L} = \left(\frac{\left(E\left\{X\right\}\right)^{\alpha}}{E\left\{X^{\alpha}\right\}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\bar{W}}\right) E\left\{X\right\} > \left(\frac{1-\alpha}{\bar{W}}\right) E\left\{X\right\} = E\left\{\bar{L}\right\}. \tag{42}$$

The inequality follows from Jensen's inequality. Switching to flexible rates stabilizes labor effort, but at a level that is higher than the mean value of $L$ under fixed rates. The sum of these two effects on the representative household's welfare is ambiguous and depends on the parameters of the model.

The effect of the deviation on consumption can be calculated from

$$\tilde{C} = X - K + \bar{B}\left(E\bar{P} - \frac{\tilde{P}}{\tilde{S}}\right).$$

Using 10 once more and after some tedious algebra one obtains

$$\tilde{C} = X - K + \frac{Cov(X, X^{\alpha})}{Var(X^{\alpha})}\frac{E\{X^{\alpha}\}}{E\{X\}}\left[E\{X\} - X\right]. \tag{43}$$

Recalling 40, one readily notices that $E\{\bar{C}\} = E\{\tilde{C}\}$: the deviation leaves the expected value of consumption unchanged. But the effect on consumption variance is unclear, although the expressions for $\bar{C}$ and $\tilde{C}$ reveal that it depends solely on $\alpha$ and the distribution of $X$.

### 4.3 Multiplicity of policy regimes: a special case

When are fixed and flexible exchange rates both equilibrium policy regimes? We can identify precise conditions for this to happen if $X$ is lognormal and $v$ is of the form 35, which we assume from now on. Then, as the appendix shows, a switch from fixed rates to flexible rates must increase the variance of consumption. The appendix also shows that the switch increases the expected cost of effort if $\alpha > \chi$, leaves it the same if $\alpha = \chi$, and reduces it otherwise. As a consequence, $\alpha \geq \chi$ is sufficient (though not necessary) for fixed rates to be an equilibrium policy.

Recall from the discussion at the end of the previous subsection that $\chi \geq \alpha$ is also sufficient for flexible rates to be an equilibrium policy. It follows that *both* flexible rates and fixed rates are equilibrium outcomes if $\alpha$ and $\chi$ are sufficiently close to each other. Hence, the fact that peso bonds are indexed does not eliminate the possibility of multiple equilibria.

If both flexible rates and fixed rates are equilibrium outcomes, the appendix shows that $\bar{B}^*$ is larger than $\tilde{B}^*$ in absolute value. That is, under fixed rates the firm issues more indexed debt and purchases more dollar assets than under flexible rates. Why? The intuition is as follows. Flexible exchange rates stabilize the price of home output, the real wage, and therefore labor effort. The home portfolio is then structured to eliminate fluctuations in consumption.

Under fixed exchange rates, by contrast, the price of home output and labor employment fluctuate. An adverse shock to exports $X$, for example, lowers $\tilde{P}$ and $\tilde{L}$ and increases leisure. Portfolios are structured *ex ante* so that when leisure rises, consumption rises too. The firm accomplishes this by issuing more indexed debt than under flexible rates, so that there is a bigger capital gain when $\tilde{P}$ falls and the real exchange rate depreciates.

Finally, the appendix shows that if both fixed and flexible exchange rates are equilibria, flexible rates again yields higher welfare. In such a case, fixed exchange rates may occur as a coordination failure: if agents expect fixed rates and arrange their portfolios accordingly, the monetary authority will validate those expectations, and social welfare will be inefficiently low. As in the case with peso bonds, social welfare would then increase if the monetary authority could commit to a policy of flexible rates, or appropriate controls on portfolio shares were enacted.

## 5 Final Remarks

We have built a model in which both portfolio composition and monetary policies are determined optimally. A key implication is that, since optimal portfolios depend on policy and vice versa, there may be more than one equilibrium policy regime. For certain parameter values and shock distributions, expectations may be self-validating: if agents expect fixed rates and arrange their portfolios accordingly, the monetary authority will indeed deliver a fixed exchange rate. This suggests that the fear of floating that allegedly obtains in many countries may be an artifact of arbitrary expectations. What the literature on fear of floating fails to take into account is that the same would happen if agents expected a policy of flexible exchange rates: assets and liabilities would be denominated in such a way as to make floating optimal for the authorities.

Which equilibrium the economy land on matters. We show that for plausible functional forms (and lognormality of the shock in the case of indexed bonds), flexible exchange rates deliver higher expected social welfare than fixed rates. Therefore, policies that anchor expectations on the flexible rates outcome –or, alternatively, induce agents to hold a portfolio that is compatible with flexible exchange rates– raise social welfare.

One limitation of the analysis is that here portfolio composition is endogenous, but only given the exogenous restrictions on the menu of assets. While we have allowed for an asset menu that included more than the usual non-contingent world currency bonds, it may be desirable and useful to derive market incompleteness from more fundamental assumptions on the environment. That remains a substantial task, however, and we can only leave it for future research.

A second limitation is that we have imposed strong restrictions on the environment and policy options. These restrictions were justified on the basis of tractability and analytical convenience, but obviously they will have to be relaxed if the model is to be the basis for more realistic policy evaluation.

# A Appendix

*Proof of claims at the end of section 3.* Assume preferences are such that

$$E\left\{-\frac{1}{2}\left(C-C^{*}\right)^{2}-\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)L^{2}\right\}. \tag{44}$$

Then, under flexible rates, 24 in the text is

$$\tilde{L}v'\left(\tilde{L}\right)=\tilde{L}^{2}=(1-\alpha)\left(C^{*}-E\left\{X\right\}+K\right)E\left\{X\right\}, \tag{45}$$

where we have used the fact that $\tilde{C}=E\left\{X\right\}-K$. Plugging 45 into 44, expected utility becomes

$$E\left\{U^{\text{flex}}\right\}=\kappa\left(C^{*}-E\left\{X\right\}+K\right)E\left\{X\right\}-\Gamma, \tag{46}$$

where $\kappa=\frac{1}{2}\left[1-(1-\alpha)\left(\frac{\theta-1}{\theta}\right)\right]>0$ and $\Gamma=\frac{1}{2}\left(C^{*}-E\left\{X\right\}+K\right)\left(C^{*}+K\right)>0$.

Under fixing, given that $\bar{C}=X-K$, wage setting equation 20 in the text can be written as

$$E\bar{L}^{2}=(1-\alpha)E\left\{\left(C^{*}-X+K\right)X\right\}.$$

Using this in the utility function 44 yields

$$E\left\{U^{\text{fix}}\right\}=\kappa E\left\{\left(C^{*}-X+K\right)X\right\}-\Gamma. \tag{47}$$

Comparing 46 and 47 we see that $EU^{\text{flex}}>EU^{\text{fix}}$ requires

$$\left(C^{*}-E\left\{X\right\}+K\right)E\left\{X\right\}>E\left\{\left(C^{*}-X+K\right)X\right\}$$

or

$$\left(E\left\{X\right\}\right)^{2}<E\left\{X^{2}\right\}.$$

which always holds for non-degenerate $X$.

*Proof of claim at the end of subsection 4.1.* A switch from flex to fix implies an expected cost of labor effort of

$$\begin{aligned}
&Ev(\bar{L})=\phi E\bar{L}^{1+\chi}\\
=\ &\phi E\left[\tilde{L}\frac{X}{E\left\{X\right\}}E\left\{\bar{P}\right\}\right]^{1+\chi} \quad \text{by 34}\\
=\ &\phi\tilde{L}^{1+\chi}\left[\frac{E\left\{\bar{P}\right\}}{E\left\{X\right\}}\right]^{1+\chi}E\left\{X^{1+\chi}\right\}\\
=\ &v(\tilde{L})\left(\frac{E\left\{X^{\alpha}\right\}}{(EX)^{\alpha}}\right)^{\frac{1+\chi}{1-\alpha}}(EX)^{-(1+\chi)}E\left\{X^{1+\chi}\right\} \quad \text{by 33.}
\end{aligned}$$

Therefore,

$$E\left\{v(\bar{L})\right\} = v(\tilde{L})E\left\{X^{1+\chi}\right\}\left(E\left\{X^{\alpha}\right\}\right)^{\frac{1+\chi}{1-\alpha}}\left(E\left\{X\right\}\right)^{-\frac{1+\chi}{1-\alpha}}.$$

Divide both sides of the last equation by $v(\tilde{L})$ and take logs:

$$\begin{aligned}
\log\left[\frac{E\left\{v(\bar{L})\right\}}{v(\tilde{L})}\right] &= \log E\left\{X^{1+\chi}\right\} + \frac{1+\chi}{1-\alpha}\log\left[E\left\{X^{\alpha}\right\}\right] - \frac{1+\chi}{1-\alpha}\log E\left\{X\right\} \\
&= \left[(1+\chi)\mu + (1+\chi)^2\frac{\sigma^2}{2}\right] + \frac{1+\chi}{1-\alpha}\left(\alpha\mu + \alpha^2\frac{\sigma^2}{2}\right) - \frac{1+\chi}{1-\alpha}\left(\mu + \frac{\sigma^2}{2}\right) \\
&= \frac{1+\chi}{1-\alpha}\left[(1-\alpha)\left(\mu + (1+\chi)\frac{\sigma^2}{2}\right) + \left(\alpha\mu + \alpha^2\frac{\sigma^2}{2}\right) - \left(\mu + \frac{\sigma^2}{2}\right)\right] \\
&= \frac{1+\chi}{1-\alpha}\frac{\sigma^2}{2}\left[(1-\alpha)(1+\chi) - (1-\alpha^2)\right] = (1+\chi)\frac{\sigma^2}{2}\left(\chi-\alpha\right),
\end{aligned}$$

where from the second equality on we have assumed that $\log X$ is normal with mean $\mu$ and variance $\sigma^2$. The claim follows.

*Proof of claims at the end of subsection 4.2.* Recall that with fixed rates, consumption is given by (40), so its variance is:

$$\begin{aligned}
Var\ \bar{C} &= Var\left[X - \frac{Cov(X, X^{\alpha})}{VarX^{\alpha}}X^{\alpha}\right] \\
&= Var(X)\left[1 - \frac{Cov^2(X, X^{\alpha})}{(VarX)(VarX^{\alpha})}\right].
\end{aligned}$$

A switch to flexible rates implies that consumption is given by (43), with variance:

$$Var\ \tilde{C} = \left[1 - \frac{\mathrm{Cov}(X, X^{\alpha})}{\mathrm{Var}\left(X^{\alpha}\right)}\frac{E\left\{X^{\alpha}\right\}}{E\left\{X\right\}}\right]^2 Var\left(X\right).$$

Hence the sign of $Var\ \tilde{C} - Var\ \bar{C}$ equals the sign of

$$\left[1 - \frac{Cov(X, X^{\alpha})}{Var\left(X^{\alpha}\right)}\frac{E\left\{X^{\alpha}\right\}}{E\left\{X\right\}}\right]^2 - \left[1 - \frac{Cov^2(X, X^{\alpha})}{(VarX)(VarX^{\alpha})}\right]. \qquad (48)$$

Assume again that $\log X \sim N(\mu, \sigma^2)$. Then,

$$EX^{\alpha} = Ee^{\alpha\log X} = e^{\mu+\sigma^2/2}$$

and so on. Some very tedious algebra then gives:

$$\frac{Cov(X, X^{\alpha})}{Var\left(X^{\alpha}\right)}\frac{E\left\{X^{\alpha}\right\}}{E\left\{X\right\}} = \frac{e^{\alpha\sigma^2} - 1}{e^{\alpha^2\sigma^2} - 1} \qquad (49)$$

and

$$\frac{Cov^2(X, X^{\alpha})}{(VarX)(VarX^{\alpha})} = \frac{\left(e^{\alpha\sigma^2} - 1\right)^2}{\left(e^{\alpha^2\sigma^2} - 1\right)\left(e^{\sigma^2} - 1\right)}$$

Change variables and define $z = e^{\sigma^2}$. Note that $z$ depends on $\sigma^2$, the variance of $\log X$, and is always greater than one. Replacing in 48 and simplifying one finds that the sign of $Var\ \tilde{C} - Var\ \bar{C}$ is given by the sign of

$$z^{1+\alpha} + z^{\alpha(1+\alpha)} + z + z^{\alpha^2} - 2z^{\alpha} - 2z^{1+\alpha^2}$$

We have not been able to find the sign of this polynomial analytically, but a graph of the above expression for $\alpha$ in $[0,1]$ and $z > 1$ makes it obvious that the sign is positive. It follows that a switch from fix to flex increases the variance of consumption.

To find the effect of the switch on the expected cost of effort, note that under fixed rates labor effort is given by:

$$\bar{L} = \frac{(1-\alpha)X}{\bar{W}},$$

while 42 says that a switch stabilizes labor effort at

$$\tilde{L} = \left[\frac{(E\{X\})^{\alpha}}{E\{X^{\alpha}\}}\right]^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\bar{W}}\right) E\{X\}.$$

Assuming 35 and replacing in the above one finds that:

$$\frac{v(\tilde{L})}{E\left\{v(\bar{L})\right\}} = \frac{(E\{X\})^{1+\chi}}{E\{X^{1+\chi}\}}\left[\frac{(E\{X\})^{\alpha}}{E\{X^{\alpha}\}}\right]^{\frac{1+\chi}{1-\alpha}}.$$

If $\log X \sim N(\mu, \sigma^2)$, after taking logs the preceding expression simplifies further to:

$$\log\left(\frac{v(\tilde{L})}{E\left\{v(\bar{L})\right\}}\right) = (1+\chi)(\alpha-\chi).$$

So the sign of $v(\tilde{L}) - E\left\{v(\bar{L})\right\}$ is equal to the sign of $(\alpha - \chi)$, as claimed in the text.

Now assume both fix and flex are equilibria and turn to the comparison of $\tilde{B}^*$ and $\bar{B}^*$. Clearly,

$$\begin{aligned}\tilde{B}^* &= K - E\left\{1/\tilde{S}\right\}\tilde{B} \\ &= K - E\left\{1/\tilde{S}\right\}\tilde{Y} = K - E\{X\},\end{aligned}$$

while

$$\begin{aligned}\bar{B}^* &= K - E\left\{\bar{P}\right\}\tilde{B} \\ &= K - E\left\{\bar{P}\right\}\frac{Cov(X,\bar{P})}{Var\bar{P}} \\ &= K - \frac{Cov(X, X^{\alpha})}{Var\,(X^{\alpha})}E\{X^{\alpha}\}.\end{aligned}$$

Hence

$$\begin{aligned}\tilde{B}^* - \bar{B}^* &= \frac{Cov(X, X^\alpha)}{Var\,(X^\alpha)} E\{X^\alpha\} - E\{X\} \\ &= E\{X\}\left[\frac{Cov(X, X^\alpha)}{Var\,(X^\alpha)}\frac{E\{X^\alpha\}}{E\{X\}} - 1\right] \\ &= E\{X\}\left[\frac{e^{\alpha\sigma^2} - 1}{e^{\alpha^2\sigma^2} - 1} - 1\right] > 0,\end{aligned}$$

where the third equality follows from 49. So $\bar{B}^* < \tilde{B}^* < 0$, as claimed in the text.

Finally, with quadratic utility, the same arguments as with nominal bonds imply that

$$(1-\alpha)E\{u'(C)X\} = E\left\{L^2\right\}$$

under both flex and fix. Then, utility under flex is

$$E\left\{U^{flex}\right\} = -\frac{1}{2}(C^* - \tilde{C})^2 - \frac{(1-\alpha)}{2}\left(\frac{\theta-1}{\theta}\right)(C^* - \tilde{C})E\{X\},$$

while that under fix we have:

$$E\left\{U^{fix}\right\} = -\frac{1}{2}E\left\{(C^* - \bar{C})^2\right\} - \frac{(1-\alpha)}{2}\left(\frac{\theta-1}{\theta}\right)E\left\{(C^* - \bar{C})X\right\}. \quad (50)$$

But note that under fixed rates

$$\bar{C} = X - K + \bar{B}(E\{X^\alpha\} - X^\alpha) = \tilde{C} + \Omega,$$

where

$$\Omega \equiv \bar{B}\left(E\{X^\alpha\} - X^\alpha\right) + (X - E\{X\})$$

and clearly $E\{\Omega\} = 0$. Replacing in 50 above yields:

$$\begin{aligned}E\left\{U^{fix}\right\} &= -\frac{1}{2}E\left\{(C^* - \tilde{C} - \Omega)^2\right\} - \frac{(1-\alpha)}{2}\left(\frac{\theta-1}{\theta}\right)E\left\{(C^* - \tilde{C} - \Omega)X\right\} \\ &= E\left\{U^{flex}\right\} - \frac{1}{2}E\left\{\Omega^2\right\} + \frac{(1-\alpha)}{2}\left(\frac{\theta-1}{\theta}\right)E\{\Omega X\}.\end{aligned}$$

So the sign of $E\left\{U^{fix}\right\} - E\left\{U^{fix}\right\}$ is the sign of

$$E\left\{\Omega^2\right\} - (1-\alpha)\left(\frac{\theta-1}{\theta}\right)E\{\Omega X\} \quad (51)$$

Now recall $\bar{B} = \frac{Cov(X^\alpha, X)}{Var X^\alpha}$. Using this one can show that

$$E\left\{\Omega^2\right\} = E\{\Omega X\},$$

so expression 51 reduces to

$$\left(1 - (1-\alpha)\left(\frac{\theta-1}{\theta}\right)\right)E\left\{\Omega^2\right\} > 0.$$

We conclude that flexing delivers higher expected welfare than fixing.

## References


[1] Aghion, P., Bacchetta, P., and A. Banerjee, "A Simple Model of Monetary Policy and Currency Crises," *European Economic Review, Papers and Proceedings* 44, 2000, 728-738.

[2] Burnside, C., Eichenbaum, M. and S. Rebelo, "Hedging and Financial Fragility in Fixed Exchange Rate Regimes," Federal Reserve Bank of Chicago, Working Paper No 99-11, 1999.

[3] Caballero, R., and A. Krishnamurthy, "Exchange Rate Volatility and the Credit Channel in Emerging Markets: A Vertical Perspective." WP, MIT, May 2004.

[4] Calvo, G. "Fixed vs. Flexible Exchange Rates: Preliminaries of a Turn-of-Millennium Rematch," mimeo, University of Maryland, 1999.

[5] Calvo, G. "Capital Market and The Exchange Rate With Special Reference to the Dollarization Debate in Latin America," *Journal of Money, Credit and Banking 33*, May 2001, 312-334.

[6] Calvo, G., and C. Reinhart, "Fear of Floating," *Quarterly Journal of Economics*, Vol. CXVIII, No. 2, May 2002.

[7] Chamon, M. and R. Hausmann, "Why Do Countries Borrow the Way they Borrow? in B. Eichengreen R. Hausmann (eds.), *Other People's Money: Debt Denomination and Financial Instability in Emerging Market Economies*. Chicago: University of Chicago Press, 2005.

[8] Chari, V.V., and P. Kehoe, "Sustainable Plans," *Journal of Political Economy* 98 (1990), 783-802.

[9] Corsetti, G. and P. Pesenti, "Self-Validating Optimum Currency Areas." NBER Working Paper No. 8783, February 2002.

[10] Hausmann, R., E. Stein and U. Panizza, "Why Do Countries Float the Way they Float? WP 418, Inter-American Development Bank, 2002.

[11] Ize, A. and E. Levy-Yeyati, "Financial Dollarization." *Journal of International Economics*, Vol. 59, 2003.

[12] Ize, A. and E. Parrado, "Dollarization, Monetary Policy and the Pass-Through," Working Paper #188, International Monetary Fund, 2002.

[13] Jeanne, O., "Why Do Emerging Economies Borrow in Foreign Currency?" mimeo, IMF Research Department, October 2001.

[14] Krugman, P. "Balance Sheets, the Transfer Problem and Financial Crises," in: International Finance and Financial Crises, P. Isard, A. Razin and A. Rose (eds.), Kluwer Academic Publishers, 1999.

[15] Morón, E. and J. Castro, "Dedollarizing the Peruvian Economy: A Portfolio Approach," Universidad del Pacifico, September 2003.

[16] Schneider, M. and A. Tornell, "Balance Sheets Effects, Bailout Guarantees and Financial Crises," mimeo, UCLA, 2000.

[17] Stein, E. H.; Hausmann, R.; Gavin, Michael; Pagés-Serra, C., "Financial Turmoil and Choice of Exchange Rate Regime," working Paper, Research Department, IADB, January 1999. Also in http://www.iadb.org/oce.

[18] Stokey, N., "Credible Public Policy," *Journal of Economics Dynamics and Control* 15 (1991), 627-56

[19] Tirole, J., "Inefficient Foreign Borrowing," *American Economic Review* 93 (2003), 1678-1702

[20] Woodford, M. *Interest and Prices.* Princeton University Press, Princeton NJ, 2004