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Asymmetric Supply Function Equilibrium with Constant Marginal Costs

Pär Holmberg

Department of Economics
Uppsala University
P.O. Box 513
SE-751 20 Uppsala
Sweden
Fax: +46 18 471 14 78

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PÄR HOLMBERG

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Asymmetric supply function equilibrium with constant marginal costs¹

Pär Holmberg²

April 18, 2005

Abstract

In a real-time electric power auction, the bids of producers consist of committed supply as a function of price. The bids are submitted under uncertainty, before the demand by the Independent System Operator has been realized. In the Supply Function Equilibrium (SFE), every producer chooses the supply function maximizing his expected profit given his residual demand. I consider a uniform-price auction with a reservation price, where demand is inelastic and exceed the market capacity with a positive probability, and firms have identical constant marginal costs but asymmetric capacities. I show that under these conditions, there is a unique SFE, which is piece-wise symmetric.

Keywords: supply function equilibrium, uniform-price auction, uniqueness, asymmetry, oligopoly, capacity constraint, wholesale electricity market

JEL codes: C62, D43, D44, L11, L13, L94

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² Department of Economics, Uppsala University, P.O. Box 513, SE-751 20 Uppsala, Sweden, Phone +46 18 471 76 35, fax: +46 18 471 14 78. E-mail: par.holmberg@nek.uu.se.

1. INTRODUCTION

It is very expensive to store electric energy, compared to the production cost. Hence, in most power systems the stored electric energy is negligible; power consumption and production have to be roughly in balance, every single minute. Most of the electric power produced is traded on the day-ahead market or with long-term agreements. But neither consumption nor production is fully predictable, so to maintain balance, adjustments have to be made in real-time. The real-time market is an important component in this process. It is an auction — often a uniform-price auction — where the independent system operator (ISO) can buy or sell additional power from the power producers. Each bidder submits a non-decreasing supply function to the real-time market. This is done before the start of the delivery period, for which the bids are valid. Hence, the imbalance — the demand by the ISO — is not known when the bids are submitted. The delivery period is typically an hour, as in California, in Pennsylvania-New Jersey-Maryland (PJM), and in the Nordic countries, or half an hour, as in Britain.

In recent studies of competition in electric power markets, the bids of oligopoly producers are often modeled as outcomes of Supply Function Equilibria (SFE). This equilibrium concept was introduced by Klemperer & Meyer [7]. Producers are assumed to submit bids simultaneously to a uniform-price auction, in a one-shot game. In the non-cooperative Nash Equilibrium, each producer commits to the supply function that maximizes its expected profit given the bids of the competitors and the properties of the uncertain demand. Klemperer & Meyer show that all smooth SFE are characterized by a differential equation, which in this paper is called the KM first-order condition.

In general, there is a continuum of possible SFE. But the presence of capacity constraints can often drastically reduce the set of SFE candidates, at least when demand is inelastic [4]. If extreme demand outcomes are allowed for, i.e. demand exceeds the total capacity with a positive probability, there is a unique symmetric equilibrium for symmetric producers with strictly convex cost functions [6]. A reservation price, i.e. a price cap, is needed to limit the equilibrium price and clear the market in case of power shortage. Risk of extreme demand outcomes, inelastic demand and reservation prices are all realistic assumptions for a real-time market [6].

Assuming symmetric producers is convenient as SFE can then be straightforwardly calculated for general cost functions [1,4,9]. But firms in electric power markets are typically asymmetric. Thus to make efficient antitrust policy and merger control possible, models that

can analyze asymmetric markets are important. In this paper it is shown that mark-ups in an asymmetric market can be considerably different from mark-ups in a symmetric market with the same market concentration, measured by the Herfindahl-Hirschman index.

Linear SFE for asymmetric firms with linear marginal costs have been analyzed by Green [5]. Baldick et al. [2] have then developed this concept to piece-wise linear SFE, which can handle asymmetric firms with affine marginal costs, i.e. linear marginal costs with asymmetric intercepts. Linear and piece-wise linear SFE are both problematic when considering capacity constraints [2].

Newbery [8] and Genc & Reynolds [4] have derived SFE for two producers with identical constant marginal costs and asymmetric capacities. Both of them consider smooth supply functions, which is routine in the SFE literature. This paper extends their work to multiple asymmetric producers. Further, this paper considers partly vertical and horizontal supply functions—i.e. binding slope constraints—and supply functions with kinks. There are three reasons why it is important to consider supply functions with horizontal or vertical segments. First, the supply function of a producer is vertical when his capacity constraint binds and horizontal when the price cap binds. Second, such segments are useful deviation strategies that can be used to rule out some SFE candidates. Third, in the considered market situation—inelastic imbalances that might be zero—, ruling out horizontal and vertical segments would rule out all SFE. The latter is also true if one rules out kinks for firms with non-binding capacity constraints. The extension complicates the analysis, as more SFE candidates have to be ruled out.

In equilibrium, a firm with non-binding capacity constraint must face a continuous elasticity of its residual demand. Otherwise the firm would have a jump in its supply function. In case of a duopoly with elastic demand and no price cap, which is studied by Newbery [8], this implies that the capacity constraint of the smaller firm must bind after a smooth transition to an inelastic demand. This ensures that kinks can be avoided in the supply function of the larger firm. However, for three firms or more, this is not necessarily true. As is shown in this paper, a continuous elasticity of the residual demand for firms with non-binding capacity constraints can also be ensured, if all of these firms discontinuously increase the elasticity of their supply at the price, for which the capacity constraint of a smaller firm binds.

Genc & Reynolds consider a duopoly with inelastic demand and a price cap. They can avoid kinks by assuming that the demand is always above some positive level, which gives them an extra free parameter. The assumption is reasonable for the day-ahead market, but not

for balancing markets, where differences to contracted power are traded and where the demand of the system operator can be both positive and negative.

This paper assumes that extreme demand outcomes occur with a positive probability. I show that there is a unique SFE for multiple producers with identical and constant marginal costs but asymmetric capacities. The unique SFE is piece-wise symmetric. Any two producers will have the same supply function, until the capacity constraint of the smaller firm binds. At this price the larger firm has a kink in its supply function. The capacity constraint of the second largest firm binds when the price reaches the price cap. The constraints of smaller firms bind below the price cap and the largest firm sells his remaining capacity at a price equal to the price cap.

The notation and assumptions are presented in Section 2. The unique SFE is derived in Section 3. It is shown that, in equilibrium, a firm only has supply with inelastic segments when his capacity constraint binds, and a supply with perfectly elastic segment when the price cap binds only. Further, all producers will offer their first units at the marginal cost. Thus, in equilibrium, the supply function of a producer must fulfill the KM first-order condition from the marginal cost up to the price where either his capacity constraint or the price cap binds. Furthermore, it is shown that an equilibrium must have the following properties: the price must be a continuous function of demand up to the price cap, no producer can face an inelastic residual demand and only one producer can have a perfectly elastic supply at the price cap. These properties give the end-condition to the system of differential equations; the capacity constraint of the second largest producer starts to bind at the price cap.

There is a unique SFE candidate that fulfills the end-condition and the KM first-order condition. It is verified that the candidate is an equilibrium, i.e. no firm will find it profitable to deviate. In Section 4, the unique SFE is numerically illustrated for the case of three asymmetric producers. In Section 5, the unique SFE is calculated for 153 firms in the Norwegian real-time market. The paper is concluded in Section 6.

2. NOTATION AND ASSUMPTIONS

The analysis here is similar to that in my previous paper [6]. But, instead of symmetric producers with strictly convex cost functions, I now consider producers with identical constant marginal costs c and asymmetric capacities. The analysis is confined to real-time and balancing markets with positive imbalances, but corresponding results can be readily derived for negative imbalances, as in [6].

There are $N \geq 2$ producers, who all have different production capacities. The bid of an arbitrary producer i , consists of a non-decreasing supply function $S_i(p)$. The aggregate supply of his competitors is denoted $S_{-i}(p)$ and the aggregate supply $S(p)$. In the original work by Klemperer & Meyer [7], the analysis is confined to twice continuously differentiable supply functions. Here, as in [6], the set of admissible supply functions is extended to allow for price intervals with inelastic supply and discontinuities in the supply function, i.e. perfectly elastic segments. Further, kinks in the piece-wise twice continuously differentiable supply functions are allowed. There are three major reasons for these extensions. First, the supply function of a firm must be inelastic when its capacity constraint binds and perfectly elastic when the price cap binds. Second, partly perfectly elastic supply functions and kinks are needed to get an equilibrium in case of constant marginal costs and asymmetric capacities. Third, partly inelastic and perfectly elastic supply functions are useful deviation strategies when ruling out equilibria. It is required that all supply functions are left-hand continuous.

Let $\bar{\varepsilon}_i$ be the capacity constraint of producer i . Without loss of generality, we can order firms according to their capacity, i.e. $\bar{\varepsilon}_1 < \bar{\varepsilon}_2 < \dots < \bar{\varepsilon}_N$. The total capacity is designated by $\bar{\varepsilon}$, i.e. $\bar{\varepsilon} = \sum_{i=1}^N \bar{\varepsilon}_i$. Denote the demand by ε and its probability density function by $f(\varepsilon)$. The density function is continuously differentiable and has a convex support set that includes zero demand and $\bar{\varepsilon}$ ³. By means of forced disconnection of consumers, the ISO ensures that demand is zero above the price cap. Thus in case $\varepsilon > \bar{\varepsilon}$, the market price equals the price cap. It is meaningless to bid above the price cap, as this would be equivalent to withholding power.

The market design is such that, the best price for the ISO is chosen — i.e. the lowest price — in case the total supply is partly inelastic and coincides with the inelastic demand.

Let $q_i(p, \varepsilon)$ be the residual demand that a producer i faces for $p < \bar{p}$. As long as the supply functions of his competitors are non-horizontal at p , his residual demand is given by:

$$q_i(p, \varepsilon) = \varepsilon - S_{-i}(p) \quad \text{if } p < \bar{p}. \quad (1)$$

If more than one producer has a supply function with a perfectly elastic segment at some price p_0 , supply rationing at this price is necessary for some demand outcomes. This is considered in previous work [6].

³ It is enough to assume that the support set includes zero demand and $\sum_{i=1}^{N-1} \bar{\varepsilon}_i + \bar{\varepsilon}_{N-1}$, but then redundant power may be withheld in equilibrium.

3. THE UNIQUE ASYMMETRIC SFE

The KM first-order condition is a differential equation derived in previous works [4,6,7]. It must be fulfilled by all elastic supply functions in any price interval, in which all supply functions are smooth, i.e. twice continuously differentiable. In Section 3.1, the system of differential equations is solved for producers with identical and constant marginal costs, and asymmetric capacities. However, the solution contains some constants, which, for the time being, are left undetermined.

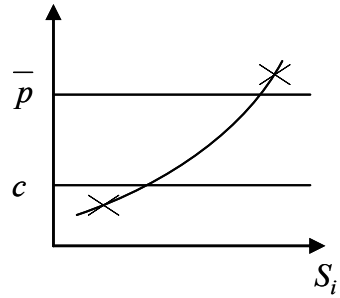
The KM first-order condition is not necessarily fulfilled by firms with binding slope constraints, i.e. supply functions with inelastic or perfectly elastic segments. However, in Section 3.2-3.5 it is shown that a firm only has inelastic segments when its capacity constraint binds and perfectly elastic segments when the price cap binds. It is a long proof, which involves the 7 propositions illustrated in Fig. 1.

In Section 3.4 (Proposition v) it is shown that all producers will offer their first unit of power at the marginal cost. This gives an initial-condition for the system of differential equations given by the KM first-order condition. Thus in equilibrium, every firm must fulfill the KM first-order condition from the marginal cost up to the price, for which either the capacity constraint of the firm or the price cap binds.

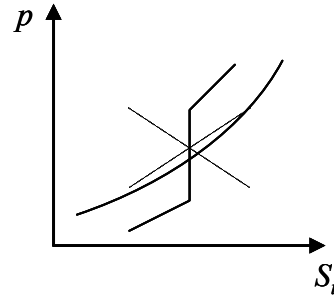
The constants in the solution of the system of differential equations are determined in Section 3.6. Due to the symmetric differential equations and the symmetric initial-condition the solution is piece-wise symmetric. Any two producers will have the same supply function until the capacity constraint of the smaller firm binds. At least two firms must have non-binding capacity constraints up to the price cap (Proposition ii and iv) and only one firm can have a perfectly elastic segment at the price cap (Proposition iii). Thus the capacity constraint of the second largest firm must start to bind at the price cap. This is the end-condition. The remaining capacity of the largest firm is sold with a perfectly elastic supply at the price cap. There is only one SFE candidate that fulfils both the KM first-order condition and the end-condition.

In Section 3.7, it is verified that the unique candidate is an equilibrium. If the competitors of a firm i follow the SFE candidate, then the profit of firm i is globally maximized for every demand outcome, if he also follows the SFE candidate. This is a sufficient condition for a SFE.

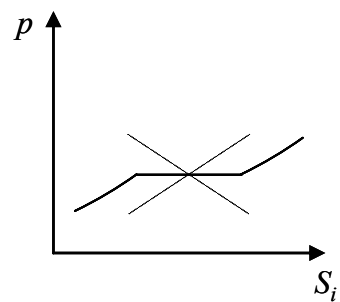
i) All market capacity is offered between c and \bar{p}



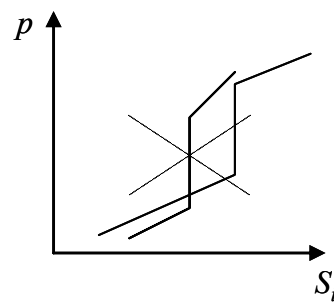
ii) A firm with elastic supply has elastic residual demand



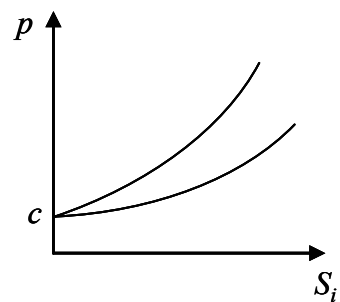
iii) No perfectly elastic segments below price cap



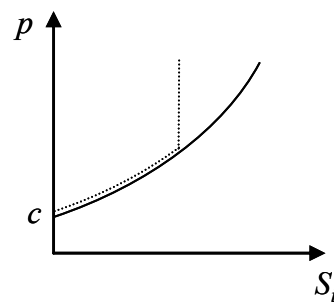
iv) No discontinuities in the equilibrium price



v) All supply functions start at c



vi) Two supply functions are identical until one becomes inelastic



vii) No inelastic segments unless the capacity constraint binds

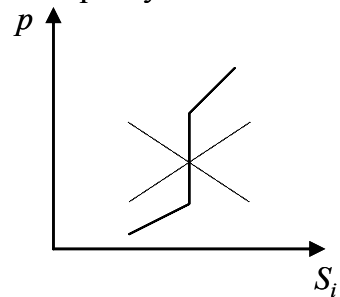


Fig. 1. Graphical illustration of Proposition i-vii, which prove necessary properties of a supply function equilibrium with constant marginal costs.

3.1. The first-order condition for smooth parts of the supply functions

Given the bids of the competitors, each producer submits his best supply function out of the class of allowed supply functions. Now consider a price interval $[p_-, p_+]$ in which all supply functions $S_i(p)$ of an equilibrium are twice continuously differentiable, i.e. no firm has kinks or perfectly elastic supply in the interval. Assume that there are $M \geq 2$ firms with elastic supply in the interval. Let E be a set of all firms belonging to this group. Denote equilibrium supply functions by $\widehat{S}_i(p)$. It can be shown that within the price interval, the supply function of each producer with elastic supply is given by the following differential equation [6]:

$$\widehat{S}_i(p) - \widehat{S}'_i(p)(p - c) = 0. \quad (2)$$

This is a special case of the KM first-order condition; the generalized version allows for general cost functions and elastic demand [7]. Firms not belonging to E have $\widehat{S}'_j(p) = 0$ in the whole range $p \in [p_-, p_+]$. Apart from the boundary conditions at p_- and p_+ , producers with inelastic supply will not influence the differential equation in (2).

In equilibrium all firms with elastic supply functions for $p \in [p_-, p_+]$ follow the KM first-order condition. Thus the supply functions of these M firms are given by a system of differential equations. The system is not linear, and is somewhat complex to solve. It is observed, however, that this particular system of differential equations can be solved in a two-step approach. First, the total supply function of the M producers with elastic supply is calculated. This elastic supply is given by $\widehat{S}_e(p) = \sum_{i \in E} \widehat{S}_i(p)$. Subsequently, the individual supply functions can be derived. We start with the first step. The supply function of each of the firms in the set E follows a differential equation as in (2). Adding all of these equations yields:

$$\sum_{i \in E} \widehat{S}_i(p) - \sum_{i \in E} \widehat{S}'_i(p)(p - c) = \widehat{S}_e(p) - (M - 1)(p - c)\widehat{S}'_e(p) = 0.$$

The differential equation is separable and the solution is:

$$S_e = \beta(p - c)^{1/(M-1)}, \quad (3)$$

where $\beta > 0$ is an arbitrary constant. For any producer i belonging to the set E , (2) can be written on the following form.

$$\widehat{S}_i(p) - (p - c)\left(\widehat{S}'_e(p) - \widehat{S}'_i(p)\right) = 0.$$

Rewriting it by means of (3) yields:

$$\widehat{S}_i(p) + \widehat{S}_i'(p)(p-c) = \frac{\beta}{M-1}(p-c)^{1/(M-1)}.$$

Thus, it follows from the rule of product differentiation that.

$$\frac{d}{dp} \{ \widehat{S}_i(p)(p-c) \} = \frac{\beta}{M-1}(p-c)^{1/(M-1)}$$

Integrating both sides yields:

$$\widehat{S}_i(p) = \frac{\beta}{M}(p-c)^{1/(M-1)} + \frac{\gamma_i}{p-c}. \quad (4)$$

Note that all of the M firms with elastic supply have the same β in the interval $[p_-, p_+]$. On the other hand, they have individual specific constants γ_i . Nevertheless, the individual solutions in (4) must add up to the aggregate solution in (3). Thus

$$\sum_{i=1}^M \gamma_i = 0. \quad (5)$$

It follows from (3) that the slope of the aggregate supply function in the interval $[p_-, p_+]$ is given by:

$$\widehat{S}_e'(p) = \frac{\beta}{M-1}(p-c)^{(2-M)/(M-1)}.$$

Thus

$$p'(\varepsilon) = \frac{1}{\frac{d\varepsilon}{dp}} = \frac{1}{\widehat{S}_e'(p)} = \frac{M-1}{\beta}(p-c)^{(M-2)/(M-1)}. \quad (6)$$

3.2. Miscellaneous properties of the supply function equilibrium

Proposition i: In equilibrium no units are offered below the marginal cost or withheld, i.e. all units are offered at a price at or below the price cap.

Proof: See proof of Proposition 1 and 6 in [6].

No power is offered below its marginal cost in equilibrium. It would be a profitable deviation to offer the power at its marginal cost. The deviation would cut the losses without reducing the contribution from profitable outcomes. Similarly, no power is withheld in equilibrium. It is better to offer this power at the price cap. The deviation will not affect the market price negatively, while the firm can increase his supply for some demand outcomes.

Proposition ii: In equilibrium there is no price interval (p_-, p_+) , where a producer with an elastic supply is facing an inelastic residual demand.

Proof: Assume that firm i has an elastic supply in (p_-, p_+) , while his residual demand is inelastic in this interval. Without losing accepted supply, he could offer all units previously offered in the range (p_-, p_+) at a price arbitrarily close to, but still below p_+ . Thus firm i would deviate.

Lemma i below proves a technicality that will be useful in later proofs.

Lemma i: An equilibrium with a smooth transition to an inelastic or perfectly elastic aggregate supply is not possible, if the price p_0 is above the marginal cost c . In the inelastic case, the result applies also for $p_0 \geq c$. In the perfectly elastic case, the result is valid both for individual firms and in aggregate.

Proof: See Appendix.

Considering the properties of the supply functions, which are required to be left-hand continuous and piece-wise smooth, Proposition ii and Lemma i together imply that:

Corollary i: For every $p > c$, one can find a sufficiently high $p_- < p$ such that all supply functions are twice continuously differentiable in the interval $[p_-, p]$. Either no firm or at least 2 firms have elastic supply in the interval.

3.3. There are no SFE with perfectly elastic segments below the price cap and no discontinuities in the equilibrium price

Proposition iii: In equilibrium, no firm has a perfectly elastic segment below the price cap. At the most one firm can have a perfectly elastic segment at the price cap.

Proof: See Appendix.

For $p > c$, the intuition of the proof is the same as for the Bertrand equilibrium, where producers undercut perfectly elastic bids of each other as long as the price exceeds the

marginal cost. If $p=c$, it can be shown that a firm will gain by increasing the price for some of the units offered at c^4 .

Lemma i shows that there are no smooth transitions to perfectly elastic supply for $p>c$. Thus, considering Proposition iii, $S_i'(p)=\infty$, can be ruled out for every $p\in(c,\bar{p})$. This leads us to the following corollary:

Corollary ii: For every $p\in(c,\bar{p})$ one can, in equilibrium, find a sufficiently large p_- and a sufficiently low p_+ such that all supply functions are twice continuously differentiable in the intervals $[p_-,p]$ and $[p,p_+]$. Further, all supply functions $S_i(\cdot)$ are continuous at p .

Proposition iv: There are no discontinuities in the equilibrium price.

Proof: See proof of Proposition 4 in [6].

Assume that there is a discontinuity in the equilibrium price at $\varepsilon_L\leq\bar{\varepsilon}$, where the price jumps from p_L to p_U , i.e. the aggregate supply is inelastic in this price interval. Then any producer with bids just below p_L can increase his expected profit by deviating. With a slight sales reduction, he can significantly increase the price for some units offered at and just below p_L and offer them close to p_U instead. Isolated prices for which all supply functions are inelastic are ruled out by Lemma i. Thus Proposition ii and iv imply that:

Corollary iii: From the lowest bid to the price cap, there must be at least two firms with elastic supply in each price interval. Further, $p'(\varepsilon)$ is finite for all demand outcomes.

3.4. Every firm will offer their first unit of power at the marginal cost

In Lemma ii (below) it is shown that at least two producers will offer their first unit of power at $p=c$. This result is then used in Proposition v, which proves that all producers must offer their first unit of power at $p=c$. The intuition of this result is that the bid of the first unit is never price-setting for other units of a firm. Thus the first unit is sold under Bertrand competition.

⁴ Note that $\lim_{p\rightarrow c} S_i'(p)=\infty$ has not been ruled out in equilibrium.

Lemma ii: In equilibrium, the first unit of power is offered at c .

Proof: Denote the lowest offer in the total supply by p^* . Assume that $p^* > c$. According to Proposition iii, only one producer can, in equilibrium, have a supply function with a perfectly elastic segment at the price cap. Thus $c < p^* < \bar{p}$. There are three implications from Section 3.3. First, perfectly elastic segments at p^* can be excluded, according to Proposition iii. Second, Corollary ii implies that a price p_+ can be found such that all producers have twice continuously differentiable supply functions in the interval $[p^*, p_+]$. Third, according to Corollary iii, at least two firms have elastic supply in the whole interval, if p_+ is sufficiently small. The aggregate supply of the firms with elastic supply is given by (3). Thus $p^* > c$ can be excluded as it would imply $S_e(p^*) > 0$. Proposition i thus implies that $p^* = c$.

Proposition v: In equilibrium, there must be some $p_+ > c$ such that all producers have elastic supply functions in the interval $[c, p_+]$

Proof: Now assume that there is a potential equilibrium, where producer i offers his first unit at the price $p^* > c$. Denote the aggregate supply and the equilibrium price of the potential equilibrium by $S^A(p)$ and $p^A(\varepsilon)$. It follows from Lemma ii that $p^A(0) = c$. Now consider the following deviation. Producer i reduces the price for an infinitesimally small unit of power, so that the first unit of power is offered at the price $p \in (c, p^*)$. For each unit of deviated power, the deviation leads to the following marginal change in expected profit:

$$\int_{S^A(p)}^{S^A(p^*)} [p^A(\varepsilon) - c] f(\varepsilon) d\varepsilon. \quad (7)$$

It follows from Corollary iii that $S^{A'}(p) > 0$ for $p \in [p, p^*]$. Thus $S^A(p^*) - S^A(p) > 0$, and the deviation is profitable. Accordingly, equilibria where producer i offers his first units of power at a price $p^* > c$ can be eliminated.

Supply functions with a perfectly elastic segment at c are excluded by Proposition iii. Thus the first unit of power of producer i must be offered with an elastic supply function in some interval $[c, p_+]$. This is true for all producers. Thus a sufficiently small p_+ can be chosen, such that all producers have elastic supply functions in the interval $[c, p_+]$

3.5. Each producer has an elastic supply, unless his capacity constraint binds

In this section it is shown that a firm has inelastic segments in its supply function only when its capacity constraint is binding. A similar result is proven by Baldick & Hogan for strictly elastic supply and general cost functions [3]. The intuition behind this result is somewhat involved. Assume that no producer with a non-binding capacity constraint has inelastic supply below p_L . Producer i and possibly some of his competitors starts to be inelastic just above p_L . According to Corollary iii, there must be at least two producers with elastic supply functions that follow the KM first-order condition just above p_L . Denote this set of elastic supply functions by \mathcal{S} . It follows from the KM first-order condition that producers with an elastic supply must face a continuous elasticity in their residual demand. Thus to compensate for the switch to an inelastic supply by producer i , the elasticity of supply functions in \mathcal{S} must increase discontinuously at p_L . This discontinuously increases the elasticity of the residual demand of producer i at p_L . Thus he wants to sell more just above p_L . Accordingly he deviates, unless his capacity constraint binds.

To accomplish the proof, it is first shown that any two firms have identical supply functions up to the price, at which either of them has an inelastic segment.

Proposition vi: In equilibrium, any two producers have identical supply functions in the interval $[c, p]$, where $p < \bar{p}$, if neither of them have supply functions with inelastic segments in this interval.

Proof: Without loss of generality, denote the two producers by 1 and 2. According to Proposition iii, neither of the two producers have perfectly elastic segments in the interval $[c, p]$. Further, it is assumed that they do not have inelastic segments in the interval either. Thus the supply functions of both producers follow the KM first-order condition in the whole interval. The number of competitors with elastic supply may change in the interval, but still the supply functions of firm 1 and 2 are given by piece-wise solutions of the type in (4) in the whole interval $[c, p]$. Proposition v implies that the two firms have the same initial condition $S_i(c) = 0$ and according to Corollary ii both supply functions are continuous. Thus firm 1 and 2 will have identical γ_i in the whole interval $[c, p]$. Hence, the two firms have identical supply functions in this interval.

Proposition vii: There is no equilibrium, where the supply function of a producer is inelastic in an interval $p \in [p_L, p_U]$, where $c \leq p_L < p_U \leq \bar{p}$, unless his capacity constraint is binding.

Proof: Assume that there is an equilibrium where producer i has an inelastic supply in the interval $p \in [p_L, p_U]$, even if his capacity constraint does not bind. Denote the equilibrium by the superscript B . Assume that $c \leq p_L < p_U \leq \bar{p}$ and that no firm has an inelastic supply below p_L , unless its capacity constraint is binding. Thus Proposition vi implies that all producers with a non-binding capacity constraint must have identical supply functions in the interval $[c, p_L]$

According to Corollary iii, there must, for every price $p \in [p_L, p_U]$, be at least two producers — not necessarily the same in the whole interval—with elastic supply. Thus for a subinterval $[p_I, p_{II}] \subseteq [p_L, p_U]$, where a firm $j \neq i$ has an elastic supply function, its supply function must satisfy (2), i.e.

$$S_j^B(p) - S_{-j}^{B'}(p)(p - c) = 0 \quad \forall p \in [p_I, p_{II}]$$

As producer i has an inelastic supply in the interval $[p_L, p_U]$, it follows that $S_{-i}^{B'}(p) > S_{-j}^{B'}(p)$ for $\forall p \in (p_I, p_{II}]$. Further, as $S_j^B(p_L) = S_i^B(p_L)$, $S_j^B(p) > S_i^B(p)$ for $\forall p \in (p_I, p_{II}]$. Thus

$$S_i^B(p) - S_{-i}^{B'}(p)(p - c) < 0 \quad \forall p \in (p_I, p_{II}] \quad (8)$$

Now consider the following deviation for producer i . He decreases the price for an infinitesimally small unit of power previously offered at the price p_U and offers it at p_L instead. The supply function is unchanged above p_U and below p_L . Per unit of deviated power, the deviation leads to the following change in expected profit:

$$\Delta E(\pi_i) = \int_{S^B(p_L)}^{S^B(p_U)} \left\{ [p^B(\varepsilon) - c] - S_i^B(p_L) p^{B'}(\varepsilon) \right\} f(\varepsilon) d\varepsilon. \quad (9)$$

The first term is due to increased sales and the second term due to the reduced price in the demand interval. As the supply of producer i is inelastic in the studied interval, (9) can be rewritten on the following form:

$$\Delta E(\pi_i) = \int_{S^B(p_L)}^{S^B(p_U)} \left\{ [p^B(\varepsilon) - c] - \frac{S_i^B(p_L)}{S_{-i}^B(p_L)} \right\} f(\varepsilon) d\varepsilon.$$

There is a producer with an elastic supply, such as producer j , at each $p \in [p_L, p_U]$. Thus it follows from (8) that $\Delta E(\pi_i) > 0$. Thus there is a profitable deviation. In equilibrium, firm i cannot have an inelastic segment in the interval $[p_L, p_U]$, unless his capacity constraint binds.

Proposition vii rules out inelastic segments in the supply of a firm unless its capacity constraint binds. Thus the following can be concluded by means of Proposition iii, v and vi:

Corollary iv: The supply of each firm is elastic, without perfectly elastic segments, follows the KM first-order condition and is identical to the supply of larger firms, from the marginal cost up to the price, at which its capacity constraint or the price cap binds.

3.6. A unique SFE candidate that fulfills the necessary conditions

Let p_i denote the price at which the capacity constraint of firm i starts to bind. Recall that $\bar{\varepsilon}_1 < \bar{\varepsilon}_2 < \dots < \bar{\varepsilon}_N$. Thus Proposition iii and Corollary iii and iv together imply that:

Corollary v: The largest firm has a perfectly elastic supply at the price cap and $c < p_1 < \dots < p_{N-1} = \bar{p}$.

The end-condition $p_{N-1} = \bar{p}$ is important; later on it will single out the unique equilibrium. Now consider the first price-interval $[c, p_1]$, where all producers have non-binding capacity constraints. As all have identical supply functions in the interval, it follows from (4) and (5) that all firms have $\gamma_i = 0$ in the interval. Thus (4) yields:

$$S_j(p) = \frac{\beta_1 (p - c)^{1/(N-1)}}{N} \quad \text{for } j=1, 2, \dots, N. \quad (10)$$

The subscript 1 of β is used to indicate that the constant is valid for the first price interval.

In the next price-interval $[p_1, p_2]$, there are $N-1$ remaining producers with non-binding capacity constraints. Following the line of argument used for the first interval, one realizes that $\gamma_i = 0$ also in this interval. Accordingly,

$$S_j(p) = \frac{\beta_2(p-c)^{1/(N-2)}}{N-1}, \text{ if } p \in [p_1, p_2] \text{ and } j = 2 \dots N.$$

Analogously, the solution for the price interval $[p_{n-1}, p_n]$ is:

$$S_j(p) = \frac{\beta_n(p-c)^{1/(N-n)}}{N-n+1}, \text{ if } p \in [p_{n-1}, p_n] \text{ and } j = n \dots N, \quad (11)$$

where $n \in [1, N]$ and $p_0 = c$. The latter is relevant for $n=1$. Combining the end-condition $p_{N-1} = \bar{p}$ with (11) yields:

$$\beta_{N-1} = \frac{2\bar{\varepsilon}_{N-1}}{p-c}. \quad (12)$$

Thus β_{N-1} can be uniquely determined. To avoid discontinuities in the supply functions and equilibrium price—which would violate Corollary ii and iii, respectively—the following relations must be fulfilled at the boundary between two price intervals:

$$S_j(p_n) = \frac{\beta_n(p_n - c)^{1/(N-n)}}{N-n+1} = \frac{\beta_{n+1}(p_n - c)^{1/(N-n-1)}}{N-n}.$$

Thus

$$p_n = \left(\frac{(N-n)\bar{\varepsilon}_n}{\beta_{n+1}} \right)^{N-n-1} + c \quad (13)$$

and

$$\beta_n = \frac{(N-n+1)\beta_{n+1}(p_n - c)^{1/(N-n-1)-1/(N-n)}}{N-n}. \quad (14)$$

Accordingly, starting with (12), all β_n can be uniquely determined by iterative use of (13) and (14). Thus there is only one candidate that fulfills the necessary conditions for a SFE.

3.7. The only remaining equilibrium candidate is a SFE

Consider an arbitrary producer i . Assume that his competitors follow the only remaining SFE candidate, $\tilde{S}_{-i}(p)$. Accordingly, it must be a best response of producer i to follow the

equilibrium candidate. Otherwise the candidate is not an equilibrium. To show best response, it is sufficient to show that $\tilde{S}_i(p)$ maximizes his profit for every demand outcome.

For $\varepsilon \in [\underline{\varepsilon}_i, \bar{\varepsilon}]$ there is—given $\tilde{S}_i(p)$ —some sufficiently low price $\tilde{p}(\varepsilon)$, such that the capacity constraint of producer i binds, if his last unit is offered at or below $\tilde{p}(\varepsilon)$. It is never a profitable deviation for producer i to push down the price below $\tilde{p}(\varepsilon)$, as his supply cannot increase beyond the capacity constraint. Thus for $\varepsilon \in [\underline{\varepsilon}_i, \bar{\varepsilon}]$ the best price must be in the range $p \in [\tilde{p}(\varepsilon), \bar{p}]$. For $\varepsilon \in [0, \underline{\varepsilon}_i]$ one can set $\tilde{p}(\varepsilon) = c$, as it is never optimal to sell power below its marginal cost. For $\varepsilon > \bar{\varepsilon}$, producer i will sell all of his capacity at the maximum price, as long as he does not withhold any power. Thus for these outcomes there is no profitable deviation from the equilibrium candidate.

The competitors are following the only remaining equilibrium candidate. Hence, according to Corollary iv, the competitors do not have supply functions with perfectly elastic segments below the price cap. Thus for $p \in [\tilde{p}(\varepsilon), \bar{p}]$ the residual demand of an arbitrary producer i is given by (1). Hence, for given demand and price, the profit of producer i is:

$$\pi_i(\varepsilon, p) = \underbrace{[\varepsilon - \tilde{S}_i(p)]}_{\tilde{S}_i} (p - c), \quad \text{if } p \in [\tilde{p}(\varepsilon), \bar{p}]. \quad (15)$$

Thus

$$\frac{\partial \pi_i(\varepsilon, p)}{\partial p} = \underbrace{[\varepsilon - \tilde{S}_i(p)]}_{\tilde{S}_i} - (p - c) \tilde{S}_i'(p), \quad \text{if } p \in [\tilde{p}(\varepsilon), \bar{p}]. \quad (16)$$

With left-hand derivatives, the result is valid also for $p = \bar{p}$. Consider a demand outcome $\varepsilon \leq \bar{\varepsilon}$ and a price interval $[p_{n-1}, p_n]$, where $\tilde{p}(\varepsilon) < p_n$. Let $\hat{p} = \max[\tilde{p}(\varepsilon), p_{n-1}]$. By means of (11), the following can be shown:

$$\tilde{S}_i(p) = \frac{N-n}{N-n+1} \beta_n (p-c)^{1/(N-n)} + \sum_{j=1}^{n-1} \varepsilon_j, \quad \text{if } p \in [\hat{p}, p_n] \quad (17)$$

Thus

$$\tilde{S}_i'(p) = \frac{\beta_n}{N-n+1} (p-c)^{1/(N-n)-1}, \quad \text{if } p \in [\hat{p}, p_n] \quad (18)$$

Hence, it follows from (11) that:

$$\check{S}_{-i}'(p) = \frac{\check{S}_N(p)}{p-c}, \text{ if } p \in [\hat{p}, p_n] \quad (19)$$

By means of (19), (16) can be written:

$$\frac{\partial \pi_i(p, \varepsilon)}{\partial p} = \underbrace{[\varepsilon - \check{S}_{-i}(p)]}_{\check{S}_i} - \check{S}_N(p), \text{ if } p \in [\hat{p}, p_n] \quad (20)$$

This is true for all intervals $[p_{n-1}, p_n]$, where $\tilde{p}(\varepsilon) < p_n$. Thus

$$\frac{\partial \pi_i(p, \varepsilon)}{\partial p} = \underbrace{[\varepsilon - \check{S}_{-i}(p)]}_{\check{S}_i} - \check{S}_N(p), \text{ if } p \in [\tilde{p}(\varepsilon), \bar{p}] \quad (21)$$

Accordingly, $\frac{\partial \pi_i(p, \varepsilon)}{\partial p}$ is monotonically decreasing in p within the interval $[\tilde{p}(\varepsilon), \bar{p}]$. Thus

for outcomes $\varepsilon \leq \check{S}(p_i)$, producer i maximizes his profit by choosing the price such that

$\frac{\partial \pi_i(p, \varepsilon)}{\partial p} = 0$. This is achieved by following $\check{S}_i(p)$, see (21), as $\check{S}_i(p) = \check{S}_N(p)$ for $p \leq p_i$.

For $\varepsilon > \check{S}(p_i)$, it follows that $\tilde{p}(\varepsilon) > p_i$. In this case, $\frac{\partial \pi_i(p, \varepsilon)}{\partial p} < 0$, as $\check{S}_N(p) > \bar{\varepsilon}_i$ for

$p \in [\tilde{p}(\varepsilon), \bar{p}]$. Then producer i maximizes his profit by maximizing $\frac{\partial \pi_i(p, \varepsilon)}{\partial p}$. According to

(21) this is achieved by following $\check{S}_i(p)$, as $\check{S}_i(p) = \bar{\varepsilon}_i$, if $p > p_i$.

Now, let's consider producer N and the case $\varepsilon \in (\check{S}(\bar{p}), \bar{\varepsilon}]$ ⁵. None of his competitors have supply functions with perfectly elastic segments; not even at the price cap. Thus (21) is valid for producer N also when $\varepsilon \in (\check{S}(\bar{p}), \bar{\varepsilon}]$. $\frac{\partial \pi_{NI}(\bar{p}, \varepsilon)}{\partial p}$ (the left-derivative) is positive, see (21)⁶.

However, the profit cannot be improved, as the price cannot be increased beyond the price cap. Thus producer N cannot do better than following $\check{S}_N(p)$.

⁵ Recall that supply functions are left hand continuous. Thus $\check{S}(\bar{p})$ does not include the largest firm's perfectly elastic supply at the price cap.

⁶ Recall that supply functions are left hand continuous. Thus $\check{S}_N(\bar{p})$ does not include the largest firm's perfectly elastic supply at the price cap.

The conclusion is that given $\tilde{S}_{-i}(p)$, where $i=1,2,\dots,N$, the unique equilibrium candidate $\tilde{S}_i(p)$, globally maximizes the profit for every demand outcome ε . Thus the unique equilibrium candidate is a SFE.

4. EXAMPLE 1 — THREE ASYMMETRIC PRODUCERS

Assume a market with three producers. The producers have identical and constant marginal cost c . Assume that the producers have $\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{2}$ of the total capacity $\bar{\varepsilon}$ and that $\bar{p} = 3c$. Order the producers according to their production capacity so that producer 1 has the smallest capacity.

Firms 2 and 3 are symmetric up to the price cap, where the capacity constraint of firm 2 starts to bind, i.e. $S_2(\bar{p}) = S_3(\bar{p}) = \frac{\bar{\varepsilon}}{3}$. The remaining capacity of firm 3 is sold at the price cap. Thus

$$p = \bar{p} = 3c, \text{ when } \frac{\bar{\varepsilon}}{3} \leq S_3 \leq \frac{\bar{\varepsilon}}{2}. \quad (22)$$

β_2 can be calculated from (12):

$$\beta_2 = \frac{2\bar{\varepsilon}/3}{2c} = \frac{\bar{\varepsilon}}{3c}.$$

Thus according to (11)

$$S_2(p) = S_3(p) = \frac{\bar{\varepsilon}(p-c)}{6c}, \text{ if } p \in [p_1, \bar{p}], \quad (23)$$

where p_1 is the price, for which the capacity constraint of firm 1 binds. By means of (13) it can be shown that $p_1 = 2c$. Now (14) can be used to calculate β_1 .

$$\beta_1 = \frac{3\beta_2 c^{1/2}}{2} = \frac{\bar{\varepsilon}}{2c^{1/2}}.$$

Thus according to (11)

$$S_1(p) = S_2(p) = S_3(p) = \frac{\bar{\varepsilon}}{6} \left(\frac{p-c}{c} \right)^{1/2}, \text{ if } p \in [c, p_1] \quad (24)$$

The unique SFE, which is characterized by (22)-(24), is presented in Fig. 2. All the supply functions are symmetric up to the price p_1 , where the capacity constraint of the smallest firm binds. Above this price, firm 2 and 3 have symmetric supply functions up to $\bar{p} = 3c$, where the capacity constraint of producer 2 binds. The remaining supply of producer 3 is offered with

perfect elasticity at $p = \bar{p}$. An intuitive explanation of the result in Fig. 2 is that producers with a large capacity have more market power and higher mark-ups for every percentage of their capacity. The equilibrium has similarities with the SFE derived by Newbery for a duopoly facing a linear demand [8]. In that case, the supply functions are also symmetric until the capacity constraint of the smaller firm binds.

At p_1 the elasticities of the supply functions of producer 2 and 3 increase discontinuously. This ensures that the elasticity of the residual demand of both producer 2 and 3 is continuous at p_1 , even if the supply of producer 1 is inelastic above p_1 . Thus all firms, except firm 1, have kinks in their supply functions, even if their capacity constraints are not binding. We see that supply functions of the unique equilibrium are not continuously twice differentiable, which is often assumed in the SFE literature [4,7].

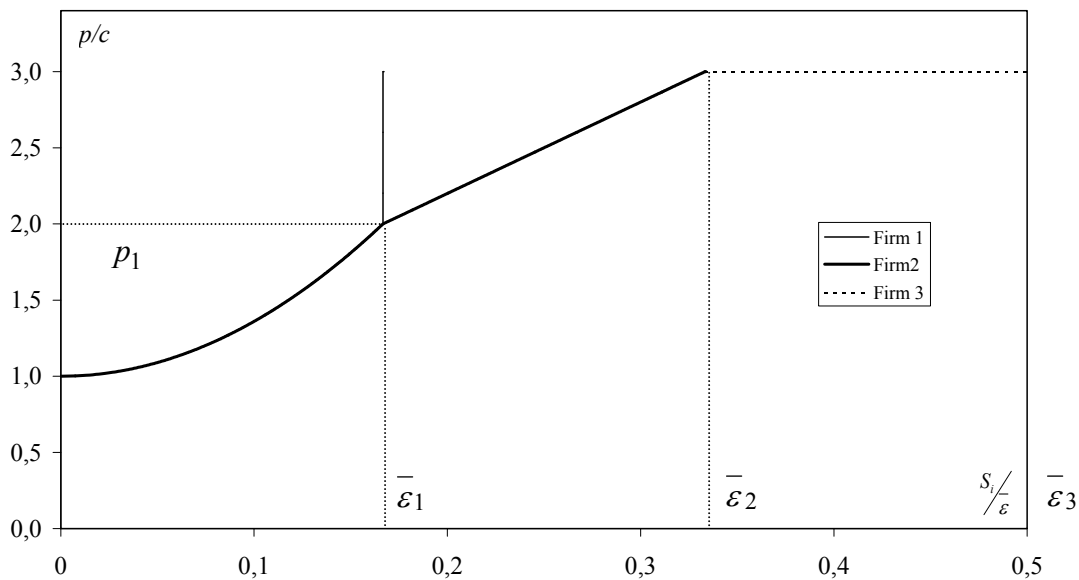


Fig. 2. *The unique supply function equilibrium for three producers with identical constant marginal cost and asymmetric capacities.*

In Fig. 2 we can note that the supply functions of all producers become perfectly elastic in the point where the price approaches c in the limit. By differentiating (10), it is straightforward to verify that this is a general result for identical and constant marginal costs. The intuition of this result is that when the producers sell their first unit, they do not have to consider that the price influences the profit from other units. Thus the first unit is sold under Bertrand competition. This also explains why the first units are sold at the marginal cost.

5. EXAMPLE 2 — THE NORWEGIAN REAL-TIME MARKET

More than 99% of the electric power production in Norway is hydroelectric, i.e. nearly all power is produced with the same technology. This makes the Norwegian real-time market suitable for the model in this paper. Now, Norway has a common electric power market with the other Nordic countries. But in this simplified example, it is assumed that Norway has a power market of its own, as before 1996. It is also assumed that all power is sold in real-time, i.e. forward and futures markets are neglected.

The marginal cost of hydropower is very small, roughly $c=50$ NOK/MWh (≈ 6 €/MWh). But water is a limited resource. Thus hydropower bids are often driven by the opportunity cost, i.e. the revenue from selling a unit of hydro power at a later day/hour. To avoid this complication, we consider an hour in the late spring, when the alternative of power production is to spill water. The price cap is 50 000 NOK/MWh (≈ 6000 €/MWh). The calculation of the SFE is based on the installed capacity of the 153 largest hydropower producers in Norway. The remaining firms and non-hydroelectric power have been neglected. The 10 largest producers in Norway and their share of the installed capacity are listed in Table 1.

The cross-ownership in Norway is extensive. Statkraft has a majority stake in Skagerak Energi and Trondheim Energiverk, Agder Energi has a majority stake in Otra kraft, and Norsk Hydro has a majority stake in Røldal-Suldal Kraft. There is, however, no available theory of how to consider cross-ownership in a SFE analysis. Thus all cross-ownerships are disregarded, although they may be important for the mark-ups.

Table 1: *Share of the installed hydroelectric capacity for the 10 largest power producers in Norway*⁷.

Company	Share of installed capacity	HHI
Statkraft	31,7%	1001,986
E-CO	7,3%	53,02816
Lyse	5,6%	31,04265
BKK	5,6%	30,92214
Norsk Hydro	4,8%	23,3467
Agder Energi	4,3%	18,37795
Skagerak Kraft	3,8%	14,52085
Otra Kraft	3,1%	9,856047
Trondheim Energiverk	2,7%	7,246727
Nord-Trøndelag Elverk	2,0%	4,20108
Rest (143 firms)	29,1%	27,2
Total	100%	1222

⁷ Data has been provided by *The Norwegian Water Resources and Energy Directorate (NVE)*.

The Herfindahl-Hirschman index (HHI) is an often used measure of the market concentration [10]. It is given by added market shares (in percentages) squared. The Norwegian electric power market has $HHI=1222$. This roughly corresponds to 8 symmetric firms, which would have $HHI=1250$. In a traditional Cournot model, with certain demand, the market mark-up would be the same for 8 symmetric firms as for the 153 asymmetric firms [10]. Thus it is interesting to compare the supply function equilibria of these two cases. It is straightforward to calculate the supply of all asymmetric firms and the 8 symmetric firms by means of the formulas in Section 3.6. In the symmetric case, the capacity constraint of all firms will start to bind at the price cap [6].

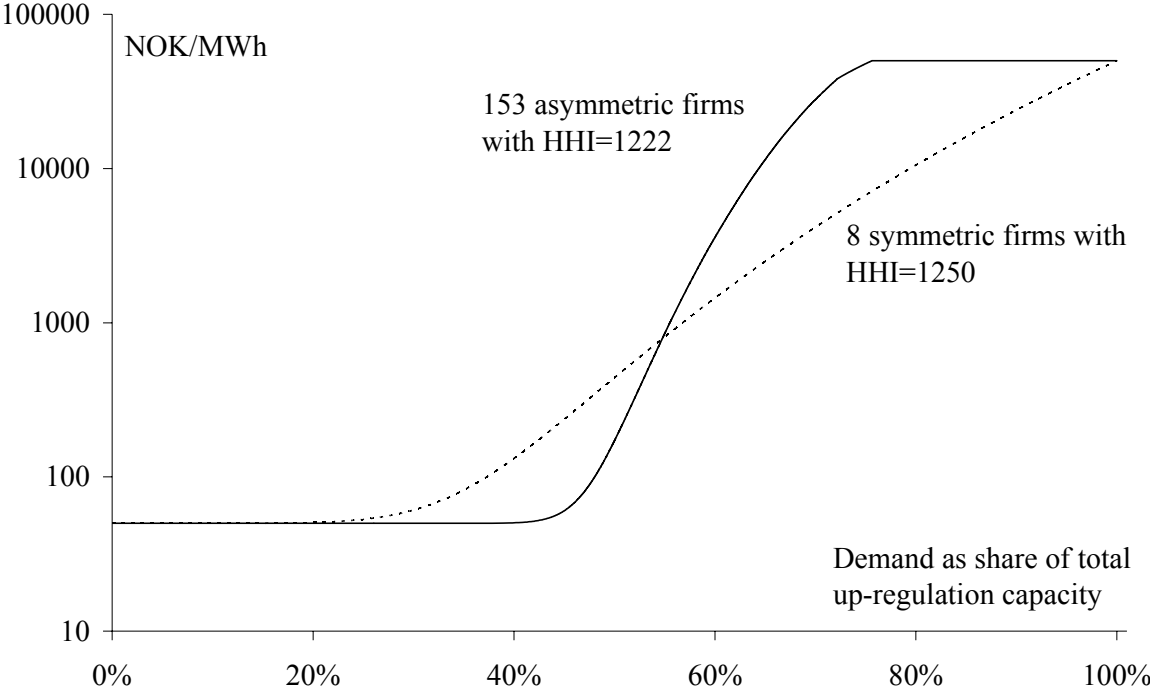


Fig. 3. The unique supply function equilibrium of the real-time market in Norway compared to a case with 8 symmetric firms.

For the asymmetric model, Fig. 3 shows that mark-ups are modest, less than 10%, for the first 44% of the capacity. For the last 52% of the capacity, mark-ups are excessive, larger than 100%. For small demands, the competition is much tougher in the asymmetric case compared to the symmetric case, as most asymmetric firms have non-binding capacity constraints. On the other hand, the competition is tougher in the symmetric case for large demands, when most asymmetric firms have binding capacity constraints. Assuming that the intuition also holds for non-constant marginal costs, it is likely that the symmetric model will overestimate

mark-ups during a summer night, when the demand in Norway is on average 30-35% of the installed capacity. On the other hand it is likely that the symmetric model will underestimate mark-ups during a winter day, when the demand is on average 60-70% of the installed capacity.

6. CONCLUSIONS

A unique Supply Function Equilibrium (SFE) is derived for an electric power market where producers have identical and constant marginal costs and production capacities are asymmetric. I assume that there is a positive probability that the demand exceeds the total market capacity. The equilibrium is piece-wise symmetric; two arbitrary producers have the same supply function until the capacity constraint of the smaller firm binds. The constraint of the producer with the second largest capacity starts to bind at the price cap. The largest firm offers its remaining capacity as a perfectly elastic supply at the price cap. The capacity constraints of smaller firms bind below the price cap.

At the price, for which the capacity constraint of a small firm starts to bind, the elasticity of the supply of larger firms will increase discontinuously. This ensures that larger firms have a continuous elasticity of their residual demand. Thus in equilibrium, all firms, but the smallest, will have kinks in their supply functions below their capacity constraint. Hence, to get an equilibrium, one cannot limit attention to smooth supply functions, which is routinely done in the SFE literature.

Compared to a symmetric SFE with the same Herfindahl-Hirschman Index (HHI), the asymmetric SFE is much more competitive for small demand outcomes, when the capacity constraints of few asymmetric firms bind. On the other hand, asymmetric SFE is less competitive for large demand outcomes, when the capacity constraints of many asymmetric firms bind.

The model is applied to the Norwegian real-time market. The model is suitable for this market, because Norway has nearly 100% hydroelectric power, which makes the constant marginal cost approximation reasonable. In the simplified example, it is assumed that Norway has a power market of its own, as before 1996, and futures and forward markets are neglected. According to the model, mark-ups are modest, below 10%, as long as the demand is below 44% of the market capacity. Excessive mark-ups, above 100%, are expected when the demand exceeds 48% of the market capacity.

7. REFERENCES

- [1] E. J. Anderson and A. B. Philpott, "Using supply functions for offering generation into an electricity market", *Operations Research*, 50, No. 3, pp. 477-489, 2002.
- [2] R. Baldick, R. Grant and E. Kahn, "Theory and Application of Linear Supply Function Equilibrium in Electricity Markets", *Journal of Regulatory Economics*, Vol. 25, No. 2, pp. 143 – 167, 2004.
- [3] R. Baldick and W. Hogan, *Capacity constrained supply function equilibrium models for electricity markets: Stability, Non-decreasing constraints, and function space iterations*, University of California Energy Institute POWER Paper PWP-089, www.ucei.berkeley.edu/ucei/PDF/pwp089.pdf, August 2002.
- [4] T. Genc & S. Reynolds, *Supply Function Equilibria with Pivotal Electricity Suppliers*, Eller College Working Paper No.1001-04, July 2004.
- [5] R. Green, "Increasing Competition in the British Electricity Spot Market", *Journal of Industrial Economics*, Vol. XLIV, No 2, pp. 205-16, 1996.
- [6] P. Holmberg, "Unique supply function equilibrium with capacity constraints", Working Paper 2004:20, Uppsala University.
- [7] P.D. Klemperer and M.A. Meyer, "Supply function equilibria in an oligopoly under uncertainty", *Econometrica*, 57, pp. 1243-1277, 1989.
- [8] D. M. Newbery, "Capacity-constrained Supply Function Equilibria: Competition and Entry in the Electricity Spot Market", Manuscript. Cambridge: Univ. Cambridge, Dept. Appl. Econm., 1991.
- [9] A. Rudkevich, M. Duckworth and R. Rosen, "Modeling electricity pricing in a deregulated generation industry: The potential for oligopoly pricing in poolco", *The Energy Journal*, 19, No. 3, pp. 19-48, 1998.
- [10] J. Tirole, *The theory of Industrial Organization*, MIT Press, Cambridge, US, 2003.

APPENDIX

Proof of Lemma i: Consider a smooth transition from the left. According to the assumed properties of the supply functions, a $p_.$ can be chosen such that all supply functions are twice continuously differentiable in the range $[p_., p^*)$. Further, $p_.$ can be chosen such that, in addition, each supply function is either monotonically increasing in the whole range or inelastic in the whole range. If all supply functions are inelastic in the interval $[p_., p^*)$, smooth transitions are not possible. Thus according to Proposition ii there must be $M \geq 2$ producers that have monotonically increasing supply functions in the range $p \in [p_., p^*)$.

It follows from (6) that one can find numbers $\underline{m} > 0$ and $\bar{m} < \infty$ such that $\underline{m} \leq p'(\varepsilon) \leq \bar{m}$, for every $p(\varepsilon) \in [p_., p^*)$, where $p_.>c$. For the upper boundary this is true also when $p \geq c$.

Thus smooth transitions from the left to a perfectly elastic aggregate supply are not possible, if $p^* > c$. This is valid for individual producers as well, as $p'(\varepsilon) = 0$ if one producer has a perfectly elastic supply. Similarly, smooth transitions from the left to an inelastic aggregate supply are not possible, if $p^* \geq c$.

Analogous proofs can be performed to rule out smooth transitions from the right

Proof of Proposition iii. The proposition can be divided into three claims:

a) In equilibrium, two or more firms cannot have supply functions with perfectly elastic segments at the same price $p^* \in (c, \bar{p}]$

b) In equilibrium, no producer has a supply function with a perfectly elastic segment at the marginal cost c .

c) In equilibrium, one firm cannot have a perfectly elastic segment at $p^* \in (c, \bar{p})$.

Proof of a) Due to Corollary i, the proof is almost identical to the proofs of Proposition 2 and 3 in [6].

Proof of b) Assume that, in equilibrium, a producer i has a perfectly elastic supply with $S_i^Q(c+) = \lim_{p \rightarrow c+} S_i^Q(p) > 0$ units of power at the constant marginal cost c . The aggregate supply of his competitors at this price is denoted by $S_{-i}^Q(c+) \geq 0$. Thus producer i may be alone with a perfectly elastic segment at c .

Assume first that the aggregate supply is elastic just above c . Then Corollary i implies that $M \geq 2$ firms are elastic just above c . The assumed properties of the supply functions ensure that a sufficiently low p_+ can be chosen such that, in equilibrium, all supply functions are twice continuously differentiable in the interval (c, p_+) , and the same $M \geq 2$ producers have an elastic supply in the price interval. The other $N-M$ producers have an inelastic supply in the whole interval. The supplies of the M producers are given by (2). The aggregate supply of this group is denoted by $S_e^Q(p)$. It follows from (3) that

$$S_e^Q(p) = \beta(p-c)^{1/(M-1)}. \quad (25)$$

Thus $S_e^Q(p)$ approaches zero as the price approaches c . Accordingly producer i and, if any, other producers with a positive supply at c cannot belong to the group with elastic supply in the interval (c, p_+) . Hence, their supply is inelastic in this interval.

Now consider the following marginal deviation of producer i . The price of an infinitesimally small unit, previously offered at c , is increased to the price $p^* \in (c, p_+]$. The marginal change in the expected profit is given by:

$$\Delta E(\pi_i) = \int_{S_i^Q(c+)}^{S_i^Q(p^*)} \left\{ S_i^Q(c+) p^Q(\varepsilon) - [p^Q(\varepsilon) - c] \right\} f(\varepsilon) d\varepsilon.$$

The first term is due to an increased price and the second term due to reduced sales in the demand interval. By means of (6), the integral can be written:

$$\begin{aligned} \Delta E(\pi_i) &= \int_{S_i^Q(c+)}^{S_i^Q(p^*)} \left\{ S_i^Q(c+) \frac{M-1}{\beta} (p^Q(\varepsilon) - c)^{(M-2)/(M-1)} - [p^Q(\varepsilon) - c] \right\} f(\varepsilon) d\varepsilon = \\ &= \int_{S_i^Q(c+)}^{S_i^Q(p^*)} [p^Q(\varepsilon) - c] \left\{ S_i^Q(c+) \frac{M-1}{\beta} (p^Q(\varepsilon) - c)^{(M-2)/(M-1)-1} - 1 \right\} f(\varepsilon) d\varepsilon. \end{aligned}$$

$(p^Q(\varepsilon) - c)^{(M-2)/(M-1)-1}$ can be made arbitrarily large for sufficiently small $p^Q > c$. Thus $\Delta E(\pi_i) > 0$ for a sufficiently small p^* . Accordingly, there are profitable deviations. Equilibria where $S_i^Q(c+) > 0$ can be ruled out, if the aggregate supply is elastic just above c .

Now consider the case where the total supply is inelastic just above c . Smooth transitions to an inelastic supply are ruled out by Lemma i. Thus if p_+ is chosen sufficiently small, the bids of all producers are now inelastic in the whole price range $p \in (c, p_+]$. Denote by ε_0 the demand outcome, for which the last unit offered at the price c is sold. Thus for the assumed equilibrium there is no contribution to the expected profit from demands below ε_0 . Now consider the following unilateral deviation of producer i . Increase the price for the $S_i^Q(c+)$ units to $p_0 \in (c, p_+)$. This measure increases the contribution to the expected profit of producer i from demand outcomes below ε_0 . The supply above p_+ is not affected. Thus the contribution to the expected profit from demands above ε_0 is not affected either. Thus the deviation increases the expected profit of producer i . Thus equilibria where a producer is offering $S_i^Q(c+) > 0$ units of power at the constant marginal cost c can also be excluded, if the aggregate supply is inelastic just above c .

Proof of c: Denote the equilibrium by the superscript W . It is assumed that $p^W(\varepsilon) = p^*$, if and only if $\varepsilon \in [\varepsilon', \varepsilon'']$. Let firm i be the producer with the perfectly elastic segment.

All firms cannot have inelastic supply just above p^* . Then firm i would deviate; he can increase the price of the perfectly elastic segment without lost sales. Further, smooth transitions to an aggregate inelastic supply are excluded by Lemma i. Thus it follows from Corollary i that at least two producers have an elastic supply just above p^* , i.e. $S_{-ir}^W'(p^*) > 0$ (the right-hand derivative). However, any competitor $j \neq i$ with an elastic bid just above p^* would find it profitable to deviate. He can slightly reduce the price of his units offered just above p^* and instead offer them just below p^* . The marginal change in the expected profit of producer j , when he deviates by one, infinitesimally small, unit is:

$$\int_{\varepsilon'}^{\varepsilon''} (p^* - c) f(\varepsilon) d\varepsilon.$$

The deviation is profitable, as $p^* > c$.

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