Electricity generation with looped transmission networks: Bidding to an ISO*

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June 15, 2004

Revised: November 20, 2004

Abstract

This paper uses a bilevel game to model markets for delivery of electrical power on looped transmission networks. It analyzes the effectiveness of an independent system operator (ISO) when generators (and, in some cases, retailers) with market power bid a single parameter of their linear supply (demand) functions to the ISO. The ISO, taking these bids at face value, maximizes welfare subject to transmission constraints. We find that equilibrium outcomes are sensitive to firms' strategy spaces. Contrary to the results of simpler models and/or published intuitions:

1. In the presence of transmission congestion and loop flows, supply function equilibria (SFE) are not bounded from above by Cournot equilibria, so Cournot outcomes may be more efficient than SFE, a difference that can be accentuated by increasing the number of rivals at a given node;

2. Allocation of transmission rights to generators can reduce efficiency; and

3. Countervailing power on the part of buyers can lower efficiency.

*The authors would like to thank Andreas Ehrenmann, Karsten Neuhoff, David Newbery, Kostas Sakellaris and two anonymous referees, for discussions and suggestions that have improved the paper; and Gordon Thompson for editing.

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1 Introduction

The transportation of electrical power on a transmission network from net generation nodes to net consumption nodes is governed by the Kirchoff Laws (Saadat [42]). Due to power flow and counterflow, the actual transmission capacity of any link in the network depends on the load flow pattern, that is, the location and quantity of any injection or withdrawal of power on the entire network. As a consequence, transmission of power is different from the transportation of an ordinary commodity in a spatial market. This difference is particularly marked when the network contains loops, and there are transmission capacity limits.[1] Reliability and security constraints, power loss, and other factors can also be important (Alvey *et al.* [2]).

This paper, derived from the first author's doctoral thesis [28], focuses on pool-type electricity markets as operated in Australia, England and Wales, New Zealand, and some parts of the United States, including the Pennsylvania-New Jersey-Maryland (PJM) electricity market [39]. In these markets, an independent system operator (ISO) determines prices by naively maximizing "social welfare" based on generators' supply function bids and on estimated, or possibly bid, demand functions, subject to transmission constraints. Limits in transmission capacity mean the ISO potentially sets a different electricity price at each node of the transmission network (sometimes referred to as nodal or locational marginal prices—Chao and Peck [8] and Hogan [26]). That is, there are multiple interlinked nodal markets, rather than a single market.

This paper presents, for two different looped-networks, numerical examples on the effectiveness of the ISO in the presence of network congestion. A simple loss-less model of electricity transmission is used (see Chao and Peck [8] and Gedra [19]). Following Berry *et al.* [5], it is assumed that cost and utility functions are quadratic,[2] and that each generator non-cooperatively bids a single parameter of their supply function (the other parameter is known).[3] Unlike Berry [5], in some cases we also allow retailers, acting for consumers, to bid one unknown parameter of their demand functions. The outcomes of these games (which we refer to as supply function equilibria (SFE) even when retailers bid demand functions) are not always intuitive, sometimes differing from simpler models of electricity transmission.

[1] On the tractability of networks without loops, see Cho [9].

[2] Other papers that use quadratic cost and utility functions include [24, 42, 21, 45, 49].

[3] Two-parameter bids allow full freedom in supply function bids and generate multiple equilibria. This is a well-known phenomenon for general supply functions in the absence of network constraints, (Klemperer and Myer [32]; see also the discussion below in §2.2 and §3.4.)

In these games, generators and buyers that bid to the ISO must solve a bilevel optimization problem. A typical bilevel problem is an optimization model whose constraints require that certain of its variables, called the "lower-level" variables, solve an optimization sub-problem that depends parametrically on the remaining "upper-level" variables. An example from economics is the leader's problem in a Stackelberg game. Our model is a multi-leader (the bidding players) single-follower (the ISO) game. With loop flows and congestion, solving for this game's equilibrium is non-trivial. Papers that model supply function bidding in electricity markets with loop flows and congestion include Berry *et al.* [5], Cardell *et al.* [7], Ehrenmann and Neuhoff [14], Hobbs *et al.* [24], Weber and Overbye [45], Xian *et al.* [49], and Younes and Ilic [48].[4] Papers that do not account for loop flows and transmission constraints include [1, 3, 9, 11, 16, 23, 26, 27, 41, 43, 46].

The paper's results are obtained using novel (at least to economics) optimization techniques and numerical methods (Hu and Ralph [29], and Hu [28]). Our approach is to replace the ISO's lower-level optimization problem by its stationary conditions (a system of equilibrium constraints, EC) converting each bidder's problem to the form of a mathematical program with EC, or MPEC. We aggregate the bidders' stationary conditions to obtain an equilibrium problem with EC (Outrata [38]), or EPEC. Although bilevel programs and EPECs are unusually challenging nonlinear programs (see, for example, Luo *et al.* [33]), they can often be successfully tackled by solving standard stationary conditions from nonlinear programming theory. This proved to be so in the present case. The problem was expressed in GAMS [6], and we searched for stationary points using PATH (Dirkse and Ferris [12], and Ferris and Munson [17]), calling these, where found, Nash stationary equilibria. In all cases, the necessary second order conditions were met, implying that the points were locally Nash equilibria (our SFE).

The paper compares different SFE with each other, and in some cases, to Cournot equilibria, and to the case where players reveal their actual supply/demand information to the ISO, the "truth-telling" outcome (call it TT). While TT is relatively easy to determine, and reporting it is not uncommon (for example, see [11, 36, 41, 44, 47]), TT should not always be taken as a normative benchmark. Rather, the relevant contrasts are between different realistic situations, which may, but need not, include TT (see discussion in Subsection 4 below). Moreover, the paper's results should be understood as counterexamples—demonstrations of the possibility of certain outcomes.

[4]Algorithms for the gaming scenarios of the models in [5, 48] are developed in [24, 45].

The paper is organized as follows. In Section 2, the bilevel game is formulated and two networks are defined: the simplest network loop, and a stylized model of the PJM. Section 3 gives a brief description of the numerical method used to solve the SFE. The main results of the paper appear in Section 4. Four comparisons are made, often providing counterexamples to standard conclusions, and showing that changes in market conditions, including bidders' strategy spaces, can significantly change the games' outcomes. The comparisons are:

§4.2. A comparison between supply function and Cournot equilibria: Examples with transmission congestion show that SFE need not be bounded from above by Cournot equilibria (contrary to the non-bilevel case—generally see Klemperer and Meyer [32], and in electricity, Green and Newbery [23]).

Increasing competition at a node where supply is constricted by congestion sharpens this difference. In the SFE cases, efficiency and consumer welfare are minimally impacted, and the profits of firms not located at the node with increased competition are raised. In the Cournot case, the equilibrium comes close to TT, dramatically raising economic efficiency and consumer welfare.

The efficiency comparisons between the SFE and Cournot outcomes are particularly striking since, in the SFE case, the ISO can set nodal prices (rather than a single price in the Cournot case), and uses information about the generators' cost functions not used in the Cournot case.

§4.3. A comparison between between (1) SFE, (2) a vertically integrated generation and transmission monopolist (so there is no need for an ISO), and (3) SFE with a generation-only monopolist: Several examples show that the granting of transmission rights to a generation monopolist reduces economic efficiency (contrary to, for example, a suggestion by Berry *et al.* [5, p. 157]).[5]

§4.4. A comparison of SFE with generator certainty and uncertainty about transmission constraints and demand: In our examples, uncertainty reduces firm profits (Oren [36] recommended exploration of this); raises consumer welfare; has an ambiguous effect

[5]For analysis directly incorporating transmission congestion contracts (Hogan [26]) into players' profit functions, see Gedra [19]; Gilbert *et al.* [20]; Joskow and Tirole [30]; Oren *et al.* [37, pp. 13 ff]; and Stoft [44]. For an example with transmission constraints see Cardell *et al.* [7].

on congestion rents (as does the form of supply function bidding); and never lowered economic efficiency, raising it in five of six cases.

§4.5. A comparison of SFE bidding by (1) generators only, and (2) generators and consumers: It is shown that bidding from consumers with market power can sharply harm efficiency, and hence may not be an attractive means of curbing market power on the generation side, contrary to Weiss [47] and King [31] and suggestions by Oren [36].[6]

We hypothesize possible explanations for these results in the relevant subsections. We are currently automating the model to allow more comprehensive testing of these hypotheses through large scale experimentation.

2 Problem formulation

This section models a bilevel game based on Berry *et al.* [5]. Subsection 2.1 presents the optimal power flow problem, in our case a quadratic program, which is solved by the ISO to determine electricity pricing and dispatch. A bidder's bilevel profit[7] maximization is given in Subsection 2.2. The final subsection gives the data specifying the two networks for our computational experiments.

2.1 The ISO's pricing and dispatch problem

We consider a loss-less DC version of power flow, following Chao and Peck [8] and Gedra [19], on a transmission network with $N+1$ nodes (sometimes called buses) labelled as $0, 1, \ldots, N$, and a set $\mathcal{L}$ of links, where the link between Node i and j is written ij.

The transmission network is managed by an ISO. Bidders (generators and, where relevant, retailers acting for consumers) have complete information about the network, the ISO's operation procedure and all other participants' cost/utility functions. They submit binding bids representing their supply or demand functions to the ISO. The ISO, taking account of the network, solves a social welfare maximization (or a social cost minimization) problem assuming the bids are truthful (as is standard in practice), announcing a dispatch for each bidder and possibly distinct prices at each node. Consumers pay generators according to the scheduled

[6]Berry *et al.* [5, p. 156] and Oren [36] raise the issue; Oren *et al.* [37, p. 12] demonstrate there may be no equilibrium with bilateral contracts.

[7]In the case of a consumer, allow profit to refer to the consumer's surplus.

dispatch and nodal prices. The announced dispatches and prices are technically feasible and clear the market according to each player's binding bid.

The existence of different prices at any two nodes implies network congestion, namely, one or more of the transmission lines is carrying electricity at its maximum capacity.

In the following exposition of the model, we assume for simplicity that there is a single generator or consumer at any Node i. However, our model is more general, allowing for multiple different bidders (generators and consumers) at a single node. In the numerical examples of the paper, generators or consumers located at the same node are assumed to have identical cost or utility functions, but when they are active bidders, each bids independently. This simplifies presentation, but provides no computational benefits, and is unlikely to impact on our qualitative conclusions.

Each player's cost or utility function is a quadratic function in quantity q_i, either cost, $A_i q_i + B_i q_i^2$ $(q_i \geq 0)$, or utility, $-A_i q_i - B_i q_i^2$ $(q_i \leq 0)$, where each A_i and B_i is assumed to be positive. At all the equilibria in the paper's numerical results, a consumer at Node i is dispatched a quantity in the range $[0, A_i/(2B_i)]$, where the consumer's utility is non-decreasing in the quantity consumed $(-q_i)$.

For expositional purposes, we focus on the case where the generator (or consumer) bids their supply (demand) function, $a_i q_i + b_i q_i^2$ $(-a_i q_i - b_i q_i^2)$, to the ISO in the form of a pair of coefficients (a_i, b_i). The ISO solves the following problem of minimizing the apparent social cost over all $N+1$ nodes in the network:

$$\begin{array}{lll} \underset{q_0,\cdots,q_N}{\text{minimize}} & \sum_{i=0}^{N} (a_i q_i + b_i q_i^2) & \\ \text{subject to} & q_0 + q_1 + \cdots + q_N = 0 & : \lambda \\ & -C_{ij} \leq \sum_{k=1}^{N} \phi_{ij,k} q_k \leq C_{ij}, \quad i < j, \; ij \in \mathcal{L}, & : \underline{\mu}_{ij}, \; \overline{\mu}_{ij} \\ & q_i \geq 0, \quad i: \text{ generator}, \quad q_i \leq 0, \quad i: \text{ consumer}, & : \nu_i \end{array} \tag{1}$$

where C_{ij} denotes the transmission limit on link ij, and $\phi_{ij,k}$ denotes the distribution factor, that is, the contribution of an injection (or withdrawal) at Node k to the link ij. The distribution factors are determined by the network's physical properties. (Note that, without loss of generality, $\phi_{ij,0}$ can be set to 0 for all links ij, hence $k = 0$ is omitted from the sum indices in the capacity constraints.) Lagrange multipliers are displayed after the corresponding constraints in (1). The optimal solution to (1) is denoted by $q = (q_0(a, b), \cdots, q_N(a, b))$, or simply $q = (q_0, \cdots, q_N)$, a vector-valued function of $(a, b) = (a_0, \cdots, a_N, b_0, \cdots, b_N)$. The Lagrange multipliers λ, $\underline{\mu}_{ij}$, $\overline{\mu}_{ij}$ and ν_i are taken to be those corresponding to the optimal solution.

The quantities q_k that optimize problem (1) commit the ISO to paying kth player a price that is consistent with their bid supply or demand function, that is:

$$p_k = a_k + 2b_k q_k \tag{2}$$

Additionally, if $q_k \neq 0$, the ISO in effect sets p_k equal to:

$$p_k = -\lambda \; - \sum_{i<j,\, ij\in\mathcal{L}} (\overline{\mu}_{ij} - \underline{\mu}_{ij})\phi_{ij,k} \tag{3}$$

(Equation (3) is obtained by considering the ISO's Karush-Kuhn-Tucker or KKT conditions reproduced in (5) below. From the last two conditions, $q_k \neq 0$ implies $v_k = 0$. By this and Equation (2), the two leftmost terms of the first equation can be replaced with p_k.)

Notice that in the absence of binding transmission constraints, p_k equals the shadow price ($-\lambda$) of the physical (as opposed to economic) requirement that electricity generated equals electricity consumed.

Some remarks on the problem (1) are in order. First, bidding rules in actual markets involve complexities not captured by the present modeling. Perhaps most importantly, in our model, as in most of the electricity SFE models we have cited, players bid continuous supply or demand functions. In contrast, in real electricity markets, the format of bids is typically a stepwise function, which is difficult to formally model. Other complexities are also common. As an example, in Australia, generators bid, sixteen hours in advance, ten steps (price/quantity pairs) for each of 48 half hours of the next day. The ISO then predispatches the generators according to forecast loads for each of the 48 half hours. However, up to five minutes before the real-time dispatch, generators are allowed to change their quantity, but not price, offers by moving their quantities up or down along the offered price stack. Our model neither captures the stepwise bids, nor the rebidding aspect of this market. Second, in practice a variety of operating constraints, must be taken into account (Alvey *et al.* [2]). Of these, reliability and security issues, though important, can be put aside when the network is in a stable state, while the effects of transmission loss are likely to be small (as found by Denton *et al.* [11], see also [2] for network losses, in a linear network).

2.2 A market participant's bidding problem

Consider the question of how a profit-maximizing player can take advantage of the market clearing procedures described above. That is, what supply or demand function should Bidder i

submit to the ISO to achieve the player's own maximal profit?

Given that, from Equation (2), Bidder i's price is $a_i + 2b_i q_i$, then its profit maximization problem is:

$$\begin{aligned} \underset{a_i, b_i}{\text{maximize}} \quad & (a_i + 2b_i q_i)q_i - (A_i q_i + B_i q_i^2) \\ \text{subject to} \quad & \underline{A}_i \leq a_i \leq \overline{A}_i \\ & \underline{B}_i \leq b_i \leq \overline{B}_i \\ & q_i \text{ such that } q = (q_0, \ldots, q_i, \ldots, q_N) \text{ solves (1)} \\ & \text{given other participants' bids } (a_{-i}, b_{-i}) \end{aligned} \tag{4}$$

recalling that $A_i q_i + B_i q_i^2$ ($q_i \geq 0$) is the generator's actual cost, and $-A_i q_i - B_i q_i^2$ ($q_i \leq 0$) is the consumer's actual utility.

The constants $\underline{A}_i, \overline{A}_i$ and $\underline{B}_i, \overline{B}_i$, assumed to satisfy

$$0 < \underline{A}_i \leq A_i \leq \overline{A}_i \quad \text{and} \quad 0 < \underline{B}_i \leq B_i \leq \overline{B}_i$$

are lower and upper bounds for a_i and b_i that are based on industry knowledge (available to all participants) and imposed by the ISO. The equilibria of games, where they exist, in which these bounds are binding, are called conjectural supply function equilibria (CSFE; see, for example, Hobbs *et al.* [24]). No bounds on bids were binding in any of the examples of this paper.

The problem just outlined is a bilevel program, where the lower-level problem is that q must solve the optimal power flow problem of the ISO, (1). Thus we refer to (1) and (4), where the latter refers to the collection of problems for each Bidder i, as a bilevel game. Looking ahead to Sections 3 and 4, our main solution procedure starts by replacing the constraint on q in (4) with the KKT conditions of (1), that is, reformulating the bilevel problem as a mathematical program with equilibrium (complementarity) constraints (MPEC).[8] We then replace all participants' MPECs by stationary conditions for these MPECs, reformulating the entire game into an EPEC.

While we examined the case where participants bid pairs of coefficients, (a_i, b_i), these generated multiple equilibria. Largely as a result (see the discussion in Subsection 3.4 below), the examples in Section 4 below further assume all participants know either A_i for all players, or B_i for all players, and the ISO requires each firm to only bid its unknown parameter, fixing the known coefficient to its true value. That is, where A_i is known, then b_i only is bid, and

[8] A complementarity constraint restricts two quantities so both are non-negative or both are non-positive, and in either case their product is zero—see the last four of the ISO-KKT constraints in problem (5).

a_i is constrained to equal A_i (or define $\underline{A}_i = A_i = \overline{A}_i$). Refer to this as the bid b game. The bid a game is correspondingly similar.

2.3 The networks modeled

We examine the following two networks:

A three-node looped network (Figure 1) which is extensively used [8, 26, 28, 19, 36, 44] to illustrate the fundamental difference between transmission networks with loop effects and ordinary commodity networks.

The distribution factors for the three-node network are $\phi_{01,1} = 2/3, \phi_{01,2} = 1/3, \phi_{02,1} = 1/3, \phi_{02,2} = 2/3, \phi_{12,1} = 1/3, \phi_{12,2} = -1/3$. The distribution factors have an intuitive explanation. Power injected at Node 1 can flow directly to Node 0 (where consumers are located) via Link 01, or by transiting Link 12 and then Link 02. If we assume all lines have the same physical parameters, so transit via the two links is twice as "hard" as by the single link, then two thirds of the power flows through Link 01 and one third through Link 12 and Link 02, and similarly, but in reverse, from Node 2.

A five-node network (Figure 2) which is taken from a PJM's tutorial course material [39].

The distribution factors of power injected at the five nodes are:

$$\begin{pmatrix} \phi_{01,1} & \phi_{01,2} & \phi_{01,3} & \phi_{01,4} \\ \phi_{02,1} & \phi_{02,2} & \phi_{02,3} & \phi_{02,4} \\ \phi_{12,1} & \phi_{12,2} & \phi_{12,3} & \phi_{12,4} \\ \phi_{13,1} & \phi_{13,2} & \phi_{13,3} & \phi_{13,4} \\ \phi_{24,1} & \phi_{24,2} & \phi_{24,3} & \phi_{24,4} \\ \phi_{34,1} & \phi_{34,2} & \phi_{34,3} & \phi_{34,4} \end{pmatrix} = \begin{pmatrix} 0.6363 & 0.3636 & 0.5454 & 0.4545 \\ 0.3636 & 0.6363 & 0.4545 & 0.5454 \\ 0.2727 & -0.2727 & 0.0909 & -0.0909 \\ 0.0909 & -0.0909 & -0.6364 & -0.3636 \\ -0.0909 & 0.0909 & -0.3636 & -0.6364 \\ 0.0909 & -0.0909 & 0.3637 & -0.3637 \end{pmatrix}$$

While we use these two networks throughout, we vary the capacities, C_{ij} on each link, and the number and type of players at each node.

3 Calculation of Nash stationary equilibria

Motivated by questions of existence and computational tractability, in Subsection 3.1 we propose two solution concepts: Nash stationary and local equilibria, which are weak variants of the classical Nash equilibrium concept.

Figure 1: A three-node network

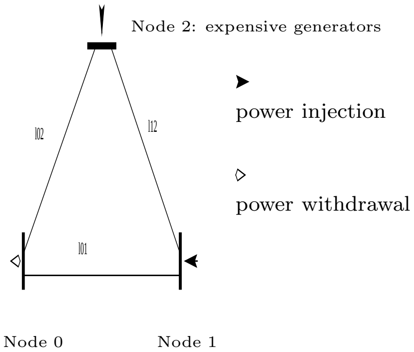
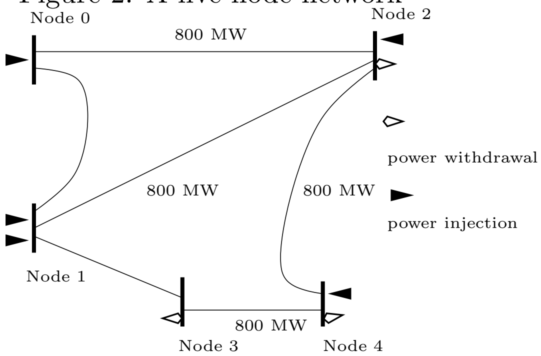


Figure 2: A five-node network



In Subsection 3.2 we formulate the bilevel problem faced by each player as an MPEC. We explain how the stationary conditions of all the players' problems yield a large KKT system, that we call All-KKT, that represents Nash stationary equilibria. Based on previous work (Fletcher *et al.* [18]), which analyzed a nonlinear programming method applied to MPECs, we anticipated that the PATH package (Dirkse and Ferris [12], and Ferris and Munson [17]) for solving KKT systems would be useful in solving All-KKT and hence in finding Nash stationary points.

Subsection 3.3 provides details on solving the All-KKT system. Preliminary numerical tests on markets over the five-node network with randomly generated cost/utility data indicate, as hoped, that finding Nash stationary points need not be computationally intractable. Checking second-order optimality conditions established that all the found points were *local* Nash equilibria.

Subsection 3.4 discusses uniqueness of the found Nash equilibria, noting the importance of restricting bids to a single parameter, a_i or b_i.

3.1 Solution concepts

If we express the dispatch q from the ISO's problem as a vector-valued function of (a, b) or, since we are thinking of the ith player's strategy, $(a_i, b_i, a_{-i}, b_{-i})$, then the bilevel game, (1) and (4), takes the form of a standard game theory model. Participant i has a profit function which can be written as $\pi_i(a_i, b_i, a_{-i}, b_{-i})$, that it seeks to maximize by choosing its strategies within prescribed bounds: $\underline{A}_i \le a_i \le \overline{A}_i$ and $\underline{B}_i \le b_i \le \overline{B}_i$. Two difficulties immediately arise. First, the profit function π_i is generally nonconcave in (a_i, b_i), which throws doubt on the existence of pure strategy Nash equilibria. Indeed, lack of existence is well-known—see, for example, Berry [5, Footnote 8, p. 143], Weber and Overbye [45], and Hu *et al.* [29]. Second, the dispatch function $q(a_i, b_i, a_{-i}, b_{-i})$ is usually nondifferentiable and cannot be readily, if at all, expressed analytically. We therefore propose a solution concept, for the bilevel game, that is weaker than the pure Nash, but is somewhat more amenable to computation.

To relax the typical concavity assumption on profit functions, we apply a local version of the Nash equilibrium concept. A *local Nash equilibrium* is a point which is a Nash equilibrium for the restricted problem that is defined by restricting the strategy space (the Cartesian product of individual participants' strategy spaces) to a sufficiently small neighborhood of this point. An even weaker equilibrium concept stems from computational practice in optimization, where algorithms are commonly designed only to find a stationary point of an optimization problem, that is, a point satisfying a first order optimality condition. Based on this, we call a point a *Nash stationary equilibrium* if the associated strategy of each participant satisfies the stationarity condition for their profit maximization problem. We have not yet explained what such a stationary condition for (4) might be; we turn to this next.

3.2 Model development

Since the ISO's problem (1) is a strictly convex quadratic program, a dispatch $q(a, b)$ solves (1) given (a, b) if and only if $q(a, b)$ is stationary for the same problem, that is, there exist multipliers corresponding to the constraints that satisfy the usual KKT conditions at $q(a, b)$ (Bazaraa [4]). Therefore, given other participants' bids (a_{-i}, b_{-i}), Bidder i's problem becomes the following MPEC:

$$
\begin{array}{ll}
\underset{a_i, b_i, q, \lambda, \underline{\mu}, \overline{\mu}, \nu}{\text{maximize}} & (a_i + 2b_i q_i)q_i - (A_i q_i + B_i q_i^2) \\
\text{subject to} & \underline{A}_i \le a_i \le \overline{A}_i, \quad \underline{B}_i \le b_i \le \overline{B}_i \\
\text{(ISO-KKT)} & \left\{
\begin{array}{l}
\text{where, for each } j = 0, \dots, N \quad \text{and } mn \in \mathcal{L} \text{ with } m < n\text{:} \\
a_j + 2b_j q_j + \lambda + \displaystyle\sum_{i<k, ik \in \mathcal{L}} \phi_{ik,j}(\overline{\mu}_{ik} - \underline{\mu}_{ik}) - \nu_j = 0 \\
q_0 + q_1 + \cdots + q_N = 0 \\
C_{mn} + \sum_{k=1}^N \phi_{mn,k} q_k \ge 0, \ \underline{\mu}_{mn} \ge 0, \ \underline{\mu}_{mn}(C_{mn} + \sum_{k=1}^N \phi_{mn,k} q_k) = 0 \\
C_{mn} - \sum_{k=1}^N \phi_{mn,k} q_k \ge 0, \ \overline{\mu}_{mn} \ge 0, \ \overline{\mu}_{mn}(C_{mn} - \sum_{k=1}^N \phi_{mn,k} q_k) = 0 \\
q_j \ge 0, \ \nu_j \ge 0, \ q_j \nu_j = 0 \quad \text{if Bidder } j \text{ is a generator} \\
q_j \le 0, \ \nu_j \le 0, \ q_j \nu_j = 0 \quad \text{if Bidder } j \text{ is a consumer}
\end{array}
\right.
\end{array}
\tag{5}
$$

where $\underline{\mu}, \overline{\mu}, q$, and ν denote vectors, the respective components of which are $\underline{\mu}_{mn}, \overline{\mu}_{mn}, q_j$, and ν_j. Here the EC are distinguished from standard constraints only by the complementarity conditions (the last two conditions of ISO-KKT). As a result, the EC may be referred to as complementarity constraints, CC, and the MPEC called an MPCC.

The game based on the all participants' problems (5), for $i = 0, \ldots, N$, gives rise to an EPEC. In particular, following Hu [28], we formulate the standard first order necessary (that is, KKT) conditions of (5) for each Bidder i. We then form a system consisting of all bidders' KKT conditions in which the ISO's KKT conditions, that appear as (ISO-KKT) in the constraints of (5), are in common (so the variables $q, \lambda, \underline{\mu}, \overline{\mu}, \nu$ are in common), hence appear only once. However, each bidder's KKT system includes different KKT multipliers for the corresponding common constraints. We call this system All-KKT; it constitutes an EPCC. We define a Nash stationary equilibrium as a solution of All-KKT—see Hu and Ralph [29] for discussion. For brevity, we omit formulation of All-KKT here; see Hu [28, Chapter 7] for details.

Formally, we are using a KKT system (and solver) in the same way that a classical game with constrained strategy sets can be modelled by aggregating the KKT systems of all bidders. However, the KKT system for (5) is not an "ordinary" KKT system because the problem is not an ordinary nonlinear program. MPECs are atypical in the class of general nonlinear programs in that standard numerical stability conditions on the constraints, termed constraint qualifications, do not hold (see Hu [28] and references therein, including the monograph Luo *et al.* [33], for a general exploration of MPECs). Nevertheless, in many situations, standard nonlinear programming methods are very effective. In particular, application of a sequential linear complementarity method, such as the path search method (Ralph [40]) that is embodied in PATH, to All-KKT, is strongly suggested by Fletcher *et al.* [18], which explains why the sequential quadratic programming method for nonlinear programs can be efficient for MPECs.

3.3 On computation

PATH is applied directly to All-KKT to find a Nash stationary point of the bilevel game. We write the All-KKT system, and subsequent checking procedures mentioned next, in the GAMS modelling language [6], and present results compiled from the WHEEL machine at The University of Wisconsin.

The algorithm can fail to find a Nash stationary point. According to our computational experience with the bilevel game (1) and (4), several factors such as the capacity limits on links, the location and number of generators and consumers, their cost/utility functions, bounds on bid variables, as well as starting points for PATH, may have significant effects on the solvability of the game. For example, using the built-in uniformly distributed random number generator of GAMS, we produced 30 cost/utility functions for a five-node network—see Table 1. We then imposed a capacity limit 500 MW on link 13 and 800 MW on other links of the five-node network in Figure 2, and required $\underline{A}_i = 0.0, \overline{A}_i = 100, \underline{B}_i = 0.0001$ and $\overline{B}_i = 2$ for all generators. We tried to find an equilibrium for the 90 games generated by allowing bids of a only, b only, and a and b together, under each of these 30 generator/consumer configurations. Out of these 90 problems, 82 were solved from starting points determined by the default action of the GAMS/solver interface (30, 27, and 25, respectively, for bid a, bid b, and jointly bid a and b).

Table 1: Location of generators and consumers and actual cost/utility functions

node	# generators	# consumers	coefficient a	coefficient b
0	1	0	uniform(30.16,30.45)	uniform(0.030,0.035)
1	2	0	uniform(30.20, 30.40)	uniform(0.035, 0.040)
2	1		uniform(33.5, 33.60)	uniform(0.06, 0.085)
		1	uniform(262.15,262.80)	uniform(0.06,0.065)
3		1	uniform(240.15,240.85)	uniform(0.065,0.068)
4	1		uniform(30.30, 30.45)	uniform(0.02, 0.04)
		1	uniform(243.45,243.50)	uniform(0.055,0.058)

GAMS default seed = 3141 is applied here

If a Nash stationary equilibrium is obtained, we check to see if a second-order sufficient condition holds, that is, we attempt to verify that the stationary point for each bidder is indeed a local maximum of its profit/utility problem (5). Given that MPECs have properties that are atypical in general nonlinear programming, checking sufficient conditions is not always straightforward—for a full description see Hu [28]. Nevertheless in all our experiments, by good luck or, in some cases, a favorable MPEC structure, all Nash stationary points were found to be local Nash equilibria.

3.4 Bidding strategies considered

We focus on the bid a and bid b games for the following reasons:

1. For bid a and bid b games, our numerical experience indicates that algorithm convergence is more or less independent of the starting points, that is, when successful, the algorithms we tested arrived at the same Nash stationary equilibrium regardless of the initial states of the game. In contrast, when bidding both a and b, given an initial point (a^0, b^0) the algorithm produces a stationary point (a, b) with a at or near a^0; this multiplicity of Nash stationary equilibria makes it difficult to compare the game outcome(s) with other market situations. Indeed, it is not hard to give an example with a continuum of local Nash equilibria, see Hu [28], which is consistent with the related analysis of Ehrenmann [13]. Multiple equilibria also arise for SFE in the absence of network constraints and when bids are applied to a unique or known demand function (Klemperer and Meyer [32]; and in electricity, Newbery [34], Green and Newbery, [23] and Newbery [35]).

2. Suppose we have a Nash stationary equilibrium point for the bid a or bid b game. It can be seen, from the KKT conditions for each participant's problem (5), that if the bounds $\underline{A}_i \le a_i \le \overline{A}_i$ and $\underline{B}_i \le b_i \le \overline{B}_i$ are inactive at that point for all participants (as is the case in all of this paper's examples), then the point also satisfies the stationary conditions for jointly bidding a and b. That is, this point must be a Nash stationary equilibrium for the bid both a and b game.

4 Market efficiency: Numerical examples

In this section, a number of comparisons of local (non-cooperative) Nash equilibria are made to illustrate possible impacts of different market environments and regulatory regimes. In all these comparisons, only generators bid strategically,[9] except in Subsection 4.5 below, where strategic bidding by generators and consumers together is examined.

If generators and consumers were to bid truthfully or the ISO was fully informed, then the ISO would be able to maximize economic efficiency. However, in the examined games, TT is not achieved due to the presence of market power and the ISO's, albeit limited, ignorance. In these circumstances, nodal prices are distorted by bidders' strategic behaviors. While TT is used as a benchmark in the analysis that follows, it is important to recognize that a difference between TT and the reported market outcomes does not indicate market failure if TT cannot

[9]This occurs when the ISO knows consumer demand, or if the utility functions used aggregate many consumers, competition among which leads to revelation of their summed demand.

be realistically achieved (Demsetz [10] and Falk [15]). That is, TT need not be a normative benchmark, but instead only the upper, perhaps unachievable, bound for economic efficiency.

Despite this, in some cases, most especially where transmission constraints are absent, sufficient competition may approximate TT, suggesting (in line with Berry *et al.* [5], and Weiss [47]) a normative preference for competition and the easing of transmission constraints. Moreover, because of the repeated nature of the bidding process, and the relative simplicity of electricity generation, engineering models of the generation process are relatively well-understood. This, especially when coupled with legal requirements on information revelation, may greatly narrow the ISO's uncertainty about cost functions. Further, requirements, supported by fines, to bid truthfully, could make TT more plausible.[10]

4.1 Definitions

Define:

- **congestion rent** = (total) payment from consumers − (total) payment to generators.
- **social welfare** = consumers' surplus + generators' surplus + congestion rent (where, as is conventional, consumers' surplus is the difference between utility and payments by consumers for energy consumption; and producers' surplus is profit).
- **dead weight loss** (DWL) = 100×(social welfare in TT − social welfare with gaming)/social welfare in TT (note the unconventional expression of DWL as a ratio).
- **welfare share** = participant's surplus/social welfare in the same scenario.
- **(welfare) share index** = welfare share in gaming/welfare share in TT (for each participant).
- **output** = total generation = (due to the loss-less network) total consumption.
- **producer (consumer) average price** = sum of all producers' receipts (consumers' payments)/total output for each given scenario.
- $\Delta(X)(Y) = 100\times(X$ with gaming $- X$ in TT$)/X$ in TT, where X stands for the aggregate surplus or average price or congestion rent or output and Y stands for generators (G) or consumers (C).

[10] Referees advise us that, in Latin America, it is common to require bids to be based on audited costs and to hold for six months; and in Chile, under certain conditions, generators are required to bid their marginal cost.

For example, Δ(price)(C) indicates the percentage change "Δ" in consumer (C) average price relative to TT.

4.2 SFE versus Cournot

In this subsection, we compare supply function and Cournot equilibria, considering the impacts of congestion, and of competition. We use the three-node network of Figure 1, with one consumer node and two generator nodes, to examine three different bidding models: bid a, bid b and Cournot games (which could be referred to as bid q). The model generally has n_1 generators at Node 1, each having a quadratic cost function $A_{1,k}q_{1,k} + B_{1,k}{q_{1,k}}^2, k = 1, \ldots, n_1$, and n_2 generators at Node 2, each having a quadratic cost function $A_{2,k}q_{2,k} + B_{2,k}{q_{2,k}}^2, k = 1, \ldots, n_2$. Aggregate consumer demand is given by $q_0 = -(A_0 - p)/(2B_0) \leq 0$, where p denotes the price for electricity and $-q_0$ the quantity of electricity in demand.

The Cournot model can be easily applied to an electricity network when net consumption only occurs at one node, since in our model a single consumer price prevails at any node. Moreover, the Cournot game is as realistic as supply function games, since it merely requires the ISO to call for quantity, rather than supply function, bids.

In the Cournot case, the market price is set equal to $A_0 + 2B_0 q_0^*$, where q_0^* equals equilibrium output. Given supply by Generator i at Node j, $j = 1, 2$, $q_{j,i}$, each Generator k at Node l ($l = 1, 2$) has the following profit maximization problem:

$$\begin{array}{ll} \underset{q_{l,k}}{\text{maximize}} & (A_0 - 2B_0(\sum_{i=1}^{n_1} q_{1,i} + \sum_{i=1}^{n_2} q_{2,i}))q_{l,k} - (A_{l,k}q_{l.k} + B_{l,k}{q_{l,k}}^2) \\ \text{subject to} & -C_{mn} \leq \sum_{j \neq 0} \phi_{mn,j} \sum_{i=1}^{n_j} q_{j,i} \leq C_{mn} \\ & q_{l,k} \geq 0 \end{array} \tag{6}$$

where $\phi_{mn,j}$ is the distribution factor of Node j to link mn and C_{mn} is the transmission limit of link mn.

Table 3 and Table 4 represent identical games, except there is no congestion in the network underlying Table 3, while, in Table 4, Link 12 has a 5 MW limit. In both cases, there is one consumer at Node 0, two cheaper generators at Node 1, and a single, more expensive, generator at Node 2 (for the cost and utility parameters see Table 2).

In the two no congestion SFE (Table 3), all generators' bids exceed their underlying cost. As a result, the ISO dispatches less electricity than in TT. Under Cournot competition, prices and firm profits are even higher, while output, overall efficiency and consumer welfare lower. That is, the bid a and bid b SFE lie between TT and the Cournot outcome, as is consistent

Table 2: Cost/utility functions and locations for Tables 3 and 4

node	# participants	(A_i, B_i)
0	1 consumer	$(90.00, 0.50)$
1	2 generators	$(10.00, 0.02)$
2	1 generator	$(10.50, 0.15)$

with non-bilevel SFE (for example, see Klemperer and Meyer [32], Green and Newbery [23], and also Berry *et al.* [5, footnote 8]).

However, when the network is congested, this need not be true. Cournot competition may be more efficient and result in greater consumer welfare—see Table 4. Moreover, this is so even though the ISO, in the bid a and bid b, as compared with the Cournot, cases, has more tools (it can set nodal prices rather than a single Cournot price), and can use more information (it makes use of all cost information excluding a single parameter of firms' cost functions, while in the Cournot case, it cannot use any cost information).

In Table 4, congestion on Link 12 leads the expensive generator at Node 2 to increase its SFE bids relative to those of the the two cheaper generators at Node 1. The impact is to substantially reduce output. The high bid causes the ISO to scale back supply from the expensive generator and the need for adequate counterflow concomitantly forces the ISO to lower dispatch from the two cheaper generators. The addition of congestion increases the expensive firm's profit more than one hundredfold in the bid a SFE, and by more than forty times in the bid b SFE. In contrast, the efficient firms' profits actually fall in the bid a case, and only rise by about ten percent in the bid b case.

This illustrates a general principle—a transmission constraint may give an otherwise disadvantaged competitor superior market power (Berry *et al.* [5])—a result that is likely driven by two forces. First, congestion allows the expensive generator to influence output from the inexpensive generators due to counterflow. Second, congestion creates a barrier between the two nodes increasing the degree to which the two cheaper generators compete with each other, and not with the more expensive generator. This gives the more expensive generator a greater ability to manipulate price because its rivals' strategies are more constrained and helps explain why the inexpensive generators do relatively poorly in the example.

The impact of competition is reinforced by a second example, illustrated in Table 5. That case is identical to the congested case in Table 4, except that there are eight instead of two

Table 3: Cournot competition vs supply function bidding in a uncongested network

<table>
<tr><th></th><th>node</th><th>bid</th><th>dispatch</th><th>Δ(dispatch)</th><th>price</th><th>Δ(price)</th><th>profit</th><th>Δ(profit)</th></tr>
<tr><td rowspan="5">TT</td><td>0</td><td>N/A</td><td>-78.50</td><td>N/A</td><td>11.50</td><td>N/A</td><td>3080.88</td><td>N/A</td></tr>
<tr><td>1</td><td>N/A</td><td>37.58 (×2)</td><td>N/A</td><td>11.50</td><td>N/A</td><td>28.24 (×2)</td><td>N/A</td></tr>
<tr><td>2</td><td>N/A</td><td>3.34</td><td>N/A</td><td>11.50</td><td>N/A</td><td>1.68</td><td>N/A</td></tr>
<tr><td colspan="3">Price(C): 11.50</td><td colspan="3">Price(G): 11.50</td><td colspan="2">Ttl surplus: 3139.04</td></tr>
<tr><td colspan="3">Surplus(C): 3080.88</td><td colspan="3">Profit(G): 58.16</td><td colspan="2">Congestion rent: 0</td></tr>
<tr><td rowspan="5">bid a</td><td>0</td><td>N/A</td><td>-77.38</td><td>-1.42</td><td>12.62</td><td>9.72</td><td>2993.78</td><td>-2.83</td></tr>
<tr><td>1</td><td>11.21</td><td>35.37 (×2)</td><td>-5.87</td><td>12.62</td><td>9.72</td><td>67.68 (×2)</td><td>139.65</td></tr>
<tr><td>2</td><td>10.63</td><td>6.64</td><td>98.45</td><td>12.62</td><td>9.72</td><td>7.47</td><td>345.32</td></tr>
<tr><td colspan="3">Δ(price)(C): 9.72</td><td colspan="3">Δ(price)(G): 9.72</td><td colspan="2">DWL: 0.08</td></tr>
<tr><td colspan="3">Δ(surplus)(C): -2.83</td><td colspan="3">Δ(profit)(G): 145.58</td><td colspan="2">Congestion rent: 0</td></tr>
<tr><td rowspan="5">bid b</td><td>0</td><td>N/A</td><td>-75.90</td><td>-3.31</td><td>14.11</td><td>22.62</td><td>2880.02</td><td>-6.52</td></tr>
<tr><td>1</td><td>0.0623</td><td>32.92 (×2)</td><td>-12.39</td><td>14.11</td><td>22.62</td><td>113.47 (×2)</td><td>301.80</td></tr>
<tr><td>2</td><td>17.93</td><td>10.50</td><td>-200.60</td><td>14.11</td><td>22.62</td><td>21.08</td><td>1157.11</td></tr>
<tr><td colspan="3">Δ(price)(C): 22.62</td><td colspan="3">Δ(price)(G): 22.62</td><td colspan="2">DWL: 0.35</td></tr>
<tr><td colspan="3">Δ(surplus)(C): -6.52</td><td colspan="3">Δ(profit)(G): 326.44</td><td colspan="2">Congestion rent: 0</td></tr>
<tr><td rowspan="5">Cournot</td><td>0</td><td>N/A</td><td>-58.23</td><td>-25.28</td><td>31.77</td><td>176.19</td><td>1695.32</td><td>-44.97</td></tr>
<tr><td>1</td><td>N/A</td><td>20.93 (×2)</td><td>-44.29</td><td>31.77</td><td>176.19</td><td>446.98 (×2)</td><td>1482.77</td></tr>
<tr><td>2</td><td>N/A</td><td>16.36</td><td>389.36</td><td>31.77</td><td>176.19</td><td>307.88</td><td>18259.33</td></tr>
<tr><td colspan="3">Δ(price)(C): 176.19</td><td colspan="3">Δ(price)(G): 176.19</td><td colspan="2">DWL: 7.71</td></tr>
<tr><td colspan="3">Δ(surplus)(C): -44.97</td><td colspan="3">Δ(profit)(G): 1966.44</td><td colspan="2">Congestion rent: 0</td></tr>
</table>

generators at Node 1. In particular, note that the total cost of supply does not change, and hence TT does not change (there are eight generators with cost functions, $10q+0.08q^2$, instead of two, with $10q + 0.02q^2$).

Here, the expensive firm at Node 2 is entirely insulated from the increase in competition. Its profits rise, marginally in the bid a SFE, and substantially in the bid b SFE, all at the expense of the cheaper generators. Indeed, the total profit of the more efficient firms at Node 1 is, for both cases, *lower* than achieved in TT (a similar result, not reproduced, is obtained when total capacity at Node 1 is quadrupled, and all other parameters are as in Table 5.)

Comparing supply function with Cournot equilibria in the experiments of Tables 4 and 5, and the unreproduced experiment, suggest that, in the presence of congestion, Cournot games may typically be more efficient than supply function bidding. In each case:

- the Cournot outcome is more efficient than either the bid a or bid b SFE; and
- increasing competition at Node 1 appears to have a progressively lesser impact on ef-

Table 4: Cournot competition can produce better results with a congested network

Congested case (5MW limit on link 12)								
	node	bid	dispatch	Δ(dispatch)	price	Δ(price)	profit	Δ(profit)
TT	0	N/A	-74.81	N/A	15.19	N/A	2798.63	N/A
	1	N/A	22.45 (×2)	N/A	10.90	N/A	10.08 (×2)	N/A
	2	N/A	29.91	N/A	19.47	N/A	134.17	N/A
	Price(C): 15.19			Price(G): 14.33			Ttl surplus: 3017.27	
	Surplus(C): 2798.63			Profit(G): 154.33			Congestion rent: 64.31	
bid a	0	N/A	-45.84	**-38.73**	44.16	**190.82**	1050.59	**-62.46**
	1	10.60	15.21 (×2)	**-32.26**	11.21	**2.87**	13.79 (×2)	**36.81**
	2	72.49	15.42	**-48.44**	77.11	**296.01**	991.44	**638.96**
	Δ(price)(C): **190.82**			Δ(price)(G): **133.00**			DWL: **15.03**	
	Δ(surplus)(C): **-62.46**			Δ(profit)(G): **560.27**			ΔCongestion rent: **668.60**	
bid b	0	N/A	-43.06	**-42.45**	46.94	**209.14**	926.92	**-66.88**
	1	0.3035	14.51 (×2)	**-35.36**	18.81	**72.60**	123.65 (×2)	**1126.28**
	2	2.3017	14.03	**-53.09**	75.08	**285.57**	876.39	**553.20**
	Δ(price)(C): **209.14**			Δ(price)(G): **159.27**			DWL: **18.05**	
	Δ(surplus)(C): **-66.88**			Δ(profit)(G): **628.09**			ΔCongestion rent: **556.26**	
Cournot	0	N/A	-56.82	**-24.05**	33.18	**118.49**	1614.33	**-42.32**
	1	N/A	17.96 (×2)	**-20.03**	33.18	**204.44**	409.73 (×2)	**3963.45**
	2	N/A	20.91	**-30.08**	33.18	**70.39**	408.64	**204.57**
	Δ(price)(C): **118.49**			Δ(price)(G): **131.60**			DWL: **5.79**	
	Δ(surplus)(C): **-42.32**			Δ(profit)(G): **695.74**			ΔCongestion rent: **-100**	

ficiency in the SFE, as compared with the Cournot outcomes. Indeed, the SFE likely converge to outcomes that are quite different from TT. In particular, the firm at Node 2, despite its cost disadvantage, appears increasingly able to act as a monopolist facing, in firms at Node 1, a competitive fringe.

We hypothesize that, under Cournot competition, the more expensive generator cannot as effectively manipulate outcomes as it receives the *same* price as its inexpensive rivals at Node 1. As a consequence, in the Cournot games, output and profit is more broadly distributed and favors efficient firms. In comparison, in the SFE games, a high bid from the expensive generator both forces lower output, and lower prices to the inexpensive generators (but not the consumer), sharply skewing profit toward the firm with market power.

In future work, we intend to test these results both analytically and through larger scale experimentation, in part made possible by automating the modeling process.

Table 5: Increasing competition reinforces the advantage of Cournot competition

Congested case (5MW limit on link 12) with 8, as compared with 2, generators at Node 1								
	node	bid	dispatch	Δ(dispatch)	price	Δ(price)	profit	Δ(profit)
TT	0	N/A	-74.81	N/A	15.19	N/A	2798.63	N/A
	1	N/A	5.61 (×8)	N/A	10.90	N/A	2.52 (×8)	N/A
	2	N/A	29.91	N/A	19.47	N/A	134.17	N/A
	(price)(C): 15.19			(price)(G): 14.33			Ttl surplus: 3017.27	
	(surplus)(C): 2798.63			(profit)(G): 154.33			(congestion rent): 64.31	
bid a	0	N/A	-45.96	**-38.57**	44.04	**190.0**	1056.27	**-62.26**
	1	10.09	3.81 (×8)	**-32.12**	10.7	**-1.85**	1.49 (×8)	**-40.84**
	2	72.73	15.48	**-48.24**	77.38	**297.38**	999.42	**644.90**
	Δ(price)(C): **190.0**			Δ(price)(G): **131.45**			DWL: **14.90**	
	Δ(surplus)(C): **-62.26**			Δ(profit)(G): **555.30**			Δ(rent): **677.72**	
bid b	0	N/A	-45.95	**-38.59**	44.05	**190.11**	1055.55	**-62.28**
	1	0.0933	3.81 (×8)	**-32.14**	10.71	**-1.72**	1.55 (×8)	**-38.66**
	2	2.1617	15.47	**-48.26**	77.40	**297.47**	999.19	**644.73**
	Δ(price)(C): **190.11**			Δ(price)(G): **131.53**			DWL: **14.91**	
	Δ(surplus)(C): **-62.28**			Δ(profit)(G): **555.43**			Δ(rent): **667.76**	
Cournot	0	N/A	-61.77	**-17.44**	28.23	**85.92**	1907.62	**-31.83**
	1	N/A	4.8 (×8)	**-14.53**	28.23	**159.05**	85.64 (×8)	**3297.14**
	2	N/A	23.38	**-21.81**	28.23	**44.99**	332.63	**147.92**
	Δ(price)(C): **85.92**			Δ(price)(G): **97.07**			DWL: **3.04**	
	Δ(surplus)(C): **-31.83**			Δ(profit)(G): **559.42**			Δ(rent): **-100.00**	

4.3 Markets with ISOs versus vertically integrated markets

In this subsection, we compare the equilibrium outcomes of five different market structures with TT. The first two are the bid a and bid b SFE for independent generators; the third is the outcome when a vertically integrated monopolist controls all generation facilities and the transmission network itself; and the fourth and fifth are the bid a and bid b SFE when generation is monopolized.

A comparison of the vertically integrated monopolist with the generation-only monopolist is of interest because it is speculated in Berry *et al.* [5, p.156] that vertical integration might increase efficiency where congestion rents are claimed by generators. Our results show this need not be so.

The vertically integrated monopolist's problem is:

$$
\begin{array}{lll}
\underset{q_0,\cdots,q_N}{\text{maximize}} & \sum\limits_{i:\text{consumer}} (A_i + 2B_i q_i)(-q_i) - \sum\limits_{i:\text{generation}} (A_i q_i + B_i q_i^2) & \\
\text{subject to} & q_0 + q_1 + \cdots + q_N = 0 & \\
 & -C_{ij} \leq \sum\limits_{k=1}^{N} \phi_{ij,k} q_k \leq C_{ij}, \quad i < j, \; ij \in \mathcal{L} & (7) \\
 & q_i \geq 0, \quad i : \text{generation}, \quad q_i \leq 0, \quad i : \text{consumer} &
\end{array}
$$

which is a strictly convex quadratic program. Therefore, any stationary point is the unique optimal solution to this problem and *vice versa*.

The generation-only monopolist controls all generating units and bids supply functions for all generating units to an ISO who, just as in the other SFE cases, seeks to maximize economic efficiency taking the submitted bids as true. That is, the generation-only monopolist's problem is:

$$
\begin{array}{lll}
\underset{a_i,b_i,i:\text{generation}}{\text{maximize}} & \sum\limits_{i:\text{generation}} (a_i + 2b_i q_i) q_i - (A_i q_i + B_i {q_i}^2) & \\
\text{subject to} & a_{li} \leq a_i \leq a_{ui}, \quad b_{li} \leq b_i \leq b_{ui} & (8) \\
 & q = (q_0, \ldots, q_i, \ldots, q_N) \text{ solves (1) with given } (a, b) &
\end{array}
$$

where $a_j = A_j, b_j = B_j$ for each consumer j.

The results reported in Table 6 are for the three-node network, without and with congestion, where Node 0 has one consumer and one generator, Node 1 has two generators and one consumer, and Node 2 has only one generator.

It can be seen from Table 6 that, unsurprisingly, there are large efficiency and consumer surplus losses under both forms of monopoly, as compared with uncoordinated bidding by independent generators, which, in turn, only differs somewhat from TT. This confirms the rationale for restructuring of the electricity industry to the extent that it promotes competition in generation. What is more interesting, in these examples, is that vertical integration tends to increase efficiency losses, and does not reduce them.

These observations are reinforced by examples from a five-node network with parameters as detailed in Table 7. The equilibria are given in Table 8. In all cases, the vertically integrated monopoly is considerably worse than the generation-only monopoly, which in turn is quite a bit worse than non-cooperative bidding by separate generators, which is only somewhat worse than TT.

In summary, the five games reported in Tables 6–8 show that vertical integration can worsen, rather than improve economic efficiency.

Table 6: Monopolistic situations vs supply function bidding

cost/utility functions and locations of bidders						
node	# generators	cost data		# consumers	utility data	
0	1	$(12.00, 0.022)$		1	$(195.00, 0.50)$	
1	2	$(10.00, 0.02)$		1	$(200.00, 0.55)$	
2	1	$(15.00, 0.05)$		0	N/A	
	Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
Uncongested case						
TT	845.25	347.51	31633.52	15.35	0	N/A
bid *a*	**63.28**	**-0.88**	**- 1.75**	**10.40**	**0**	**0.05**
bid *b*	**91.90**	**- 1.29**	**- 2.56**	**15.28**	**0**	**0.10**
VI mon	**1847.92**	**- 49.37**	**- 74.36**	**585.34**	**N/A**	**24.33**
gen mon (bid a)	**1847.56**	**−49.37**	**−74.35**	**585.23**	**0**	**24.33**
gen mon (bid b)	**1845.45**	**−49.30**	**−74.29**	**584.50**	**0**	**24.32**
Congested case with a capacity limit 15 MW on link 12						
TT	756.86	347.49	31633.76	15.35	56.14	N/A
bid *a*	**155.55**	**- 1.87**	**- 3.69**	**22.12**	**-40.83**	**0.04**
bid *b*	**668.84**	**- 8.86**	**- 16.92**	**104.93**	**61.92**	**0.78**
VI mon	**2073.38**	**- 49.43**	**- 74.43**	**586.20**	**N/A**	**24.37**
gen mon (bid a)	**2073.16**	**-49.43**	**- 74.42**	**586.09**	**-100.00**	**24.36**
gen mon (bid b)	**2072.34**	**-49.34**	**- 74.33**	**585.07**	**-100.00**	**24.30**

4.4 Effects of uncertainty

In this subsection, we examine the impact of demand and transmission constraint uncertainty on SFE. In electricity markets, both demand and transmission limits on links may fluctuate over the course of a day (in the latter instance, for example, due to the temperature of the transmission lines, which depends on the heat generated in transmission, air temperature and other factors). When a generator makes a bid that must cover different demands or network conditions, this is equivalent to bidding for different possible realizations of uncertain outcomes. Bidding for demand over time in deregulated British electricity markets is modeled in Green and Newbery [23], by modifying Klemperer and Meyer [32] who consider demand uncertainty. In the six cases we examine, uncertainty does not lower, and usually raises economic efficiency, reduces profits, and lowers consumer prices, raising consumer welfare.

We use the five-node network described in Table 7, focussing on TT and the SFE when independent generators bid a only or bid b only. Table 9 compares SFE when transmission constraints on Link 01 can take one of three values. Under certainty, generators can place

Table 7: Cost/utility functions and locations of bidders for Table 8

node	# generators	(A_i, B_i)	# consumers	utility data
0	1	(12.00, 0.010)	0	N/A
1	2	(10.00, 0.025)	0	N/A
2	1	(14.00, 0.005)	1	(100.00, 0.200)
3	0	N/A	2	(80.00, 0.100)
4	1	(20.00, 0.050)	1	(270.00, 0.500)

a different bid for each of the three cases. The two rows labelled 'bid separately' give the average outcomes of the three SFE for each of the 'bid a' and 'bid b' cases.

With uncertainty, the generators' bids are fixed for three dispatching periods of equal length regardless of the specific transmission limits on Link 01 (see rows labelled 'bid once for three periods'), and the average outcome is reported.[11]

Table 10 compares SFE when demand can take one of two values with equal probability, or alternatively there are two demand periods of equal length. The comparison is made twice—for when the network is congested and then not congested. Simple averages are again reported.

In the examined cases, uncertainty increased, or, in one case, did not reduce, market efficiency; reduced profits, so, in that sense, market power; and reduced the prices faced by consumers. The impact of transmission uncertainty on congestion rents was ambiguous (see Table 9). For the bid a game, uncertainty reduced congestion rents bringing them close to, but still above TT. In contrast, in the bid b game, uncertainty turns a large deficit as compared with TT into a large surplus. This reinforces what appears to be a general result (see the tables throughout this Section, §4), that the bid a and b games can each either raise or lower congestion rents as compared with TT.

4.5 Bidding by consumers—countervailing power?

This subsection provides some examples of strategic bidding behavior by consumers as well as generators. It has been suggested that strategic bidding from the consumer side, perhaps

[11]The certainty case can be thought of as the expected outcome when three different constraints can occur with equal probability, but the generators know in advance as to which constraint they face. In the uncertain case, the expected outcome is reported when generators must choose one bid in advance of knowing the realized constraint.

Table 8: Monopolistic situations vs supply function bidding for the five-node network

aggregate value and their changes in different operating situations						
	Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
Uncongested case with flows: 155.05 MW on link 02, 494.07 MW on link 13						
TT	3273.15	1073.22	59075.86	18.28	0	N/A
bid a	**65.82**	**-2.68**	**-3.82**	**11.67**	**0**	**0.17**
bid b	**85.75**	**-3.52**	**-5.00**	**15.33**	**0**	**0.24**
VI mon	**880.07**	**-48.28**	**-73.74**	**290.89**	**0**	**23.67**
gen mon (bid a)	**629.30**	**-48.28**	**-52.90**	**209.98**	**0**	**17.08**
gen mon (bid b)	**628.87**	**-48.26**	**-52.88**	**209.86**	**0**	**17.09**
Congested case (280MW limit on link 13, 800 MW on others)						
TT	2075.81	834.54	44707.11	32.38	11199.69	N/A
bid a	**80.36**	**-3.78**	**-4.24**	**6.69**	**0.62**	**0.27**
bid b	**111.09**	**-4.57**	**-5.26**	**8.45**	**-1.61**	**0.39**
VI mon	**1445.39**	**-33.49**	**-65.30**	**120.71**	**N/A**	**17.92**
gen mon (bid a)	**1049.96**	**-33.49**	**-37.76**	**75.03**	**-100.00**	**10.84**
gen mon (bid b)	**1049.28**	**-33.46**	**-37.73**	**74.96**	**-100.00**	**10.84**
Congested case (80MW limit on link 02, 800 MW on others)						
TT	3177.06	1067.49	58531.50	18.79	435.97	N/A
bid a	**182.12**	**-9.76**	**-13.44**	**41.27**	**34.95**	**3.10**
bid b	**319.07**	**-17.07**	**-22.45**	**71.69**	**-45.64**	**5.15**
VI mon	**909.08**	**-48.09**	**-73.57**	**280.82**	**N/A**	**23.52**
gen mon (bid a)	**650.19**	**-48.13**	**-52.55**	**202.10**	**-100.00**	**16.96**
gen mon (bid b)	**649.78**	**-48.10**	**-52.53**	**201.99**	**-100.00**	**16.96**

through retailers, can relieve market power on the generation side of the market Berry *et al.* [5], King [31], and Oren [36]. Moreover, in an experimental game, Weiss [47] concludes that "Active demand-side bidding significantly lowers market power for a given number of sellers and therefore promises to be an excellent market power mitigation strategy". In contrast, our examples (Table 11) demonstrate that strategic bidding by consumers can bring markedly worse SFE as compared with the case where consumers bid their actual demand functions (for example, if they had no market power).

In Table 11, the *gen* rows describe games when only generators bid strategically and consumers reveal their true utility functions to the ISO. In contrast, in the *gen+con* rows both generators and consumers present bids to the ISO. With the exception of the bid b case for the uncongested three-node network (the fourth and fifth rows), consumer bidding lowers economic efficiency, often sharply. Output also falls in all cases. Consumer welfare rises, but

Table 9: Under three different network conditions (the five-node network)

cost/utility functions and location of bidders as in Table 7							
Three periods with different transmission limits 160MW, 200MW, 500MW on Link 01							
		Δ(profit) (G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
bid separately	bid *a*	**96.94**	**-4.50**	**-6.14**	**17.70**	**19.52**	**0.24**
	bid *b*	**179.78**	**-6.28**	**-8.74**	**17.61**	**-60.86**	**0.65**
bid once for three periods	bid *a*	**88.36**	**-3.67**	**-5.14**	**14.94**	**3.97**	**0.17**
	bid *b*	**128.80**	**-6.21**	**-8.48**	**14.83**	**31.02**	**0.46**

Table 10: Under two different demand periods (locations of generators and their cost functions and the five-node network configuration as in Table 7 and Table 8)

The transmission limits on Link 01 is 200MW (congested)							
		Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
bid separately	bid *a*	**112.81**	**-6.13**	**-9.62**	**25.35**	**25.09**	**0.47**
	bid *b*	**207.77**	**-8.87**	**-14.33**	**25.15**	**-31.95**	**1.19**
bid once for two periods	bid *a*	**100.51**	**-5.56**	**-8.48**	**22.42**	**20.20**	**0.43**
	bid *b*	**207.66**	**-8.85**	**-14.33**	**22.12**	**-31.90**	**1.19**
The transmission limits on Link 01 is 800MW (uncongested)							
bid separately	bid *a*	**56.49**	**-2.06**	**-3.43**	**11.59**	**0.0**	**0.09**
	bid *b*	**77.14**	**-2.86**	**-4.71**	**11.60**	**0.0**	**0.14**
bid once for two periods	bid *a*	**54.50**	**-2.06**	**-3.30**	**11.19**	**0.0**	**0.08**
	bid *b*	**76.82**	**-2.82**	**-4.69**	**11.15**	**0.0**	**0.14**

Utility functions for Period 1: $(100, 0.20)$ at Node 2, $(80, 0.10)$ at Node 3, $(270, 0.5)$ at Node 4 .

Utility functions for Period 2: $(150, 0.15)$ at Node 2, $(120, 0.10)$ at Node 3, $(270, 0.45)$ at Node 4.

in most cases, not by a great deal. This is not surprising given standard models of bilateral monopoly,[12] and this casts substantial doubt on whether countervailing market power should be relied on to reduce market power. In addition, congestion rent is significantly reduced when consumers bid strategically. This would hide the need for transmission expansion and send poor signals to network investors.

In short, in terms of total output and both short run and long run economic efficiency, countervailing power from consumers may be far from a panacea in mitigating market power.

[12] Another standard result is that a Nash equilibrium may not exist for bilateral monopoly, a result demonstrated in the context of electricity in Oren *et al.* [37, p. 12].

Table 11: Only generators bidding vs both generators and consumers bidding

	whoplay	Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
Uncongested case (bidders' data and the three-node network as in Table 6)							
bid a	gen	**63.28**	**-0.88**	**-1.75**	**10.40**	**0**	**0.05**
	gen+con	**60.05**	**-1.96**	**-1.69**	**10.00**	**0**	**0.09**
bid b	gen	**91.90**	**-1.29**	**-2.56**	**15.28**	**0**	**0.10**
	gen+con	**60.05**	**-1.96**	**-1.69**	**10.00**	**0**	**0.09**
Congested case (15MW limit on Link 12) (bidders' data and the three-node network as in Table 6)							
bid a	gen	**155.55**	**-1.86**	**-3.69**	**22.12**	**-40.83**	**0.04**
	gen+con	**147.48**	**-3.47**	**-3.59**	**21.29**	**-42.35**	**0.13**
bid b	gen	**668.84**	**-8.86**	**-16.92**	**104.93**	**61.92**	**0.78**
	gen+con	**561.69**	**-17.51**	**-16.59**	**96.84**	**-14.76**	**3.10**
Uncongested case (bidders' data and the five-node network as in Table 7 and Table 8)							
bid a	gen	**65.82**	**-2.68**	**-3.82**	**11.67**	**0**	**0.17**
	gen+con	**61.19**	**-4.17**	**-3.64**	**11.03**	**0**	**0.23**
bid b	gen	**85.75**	**-3.52**	**-5.00**	**15.33**	**0**	**0.24**
	gen+con	**78.14**	**-5.79**	**-4.73**	**14.34**	**0**	**0.38**
Congested case (280MW limit on Link 13) (bidders' data and the five-node network as in Table 7 and Table 8)							
bid a	gen	**80.36**	**-3.78**	**-4.24**	**6.69**	**0.62**	**0.27**
	gen+con	**44.91**	**-8.88**	**6.54**	**-14.88**	**-40.62**	**1.20**
bid b	gen	**111.09**	**-4.57**	**-5.26**	**8.45**	**-1.61**	**0.40**
	gen+con	**76.90**	**-12.27**	**9.88**	**-23.24**	**-64.86**	**2.16**

5 Conclusion

Dramatic reforms in markets for electricity generation have been undertaken in a number of countries around the world, typically by requiring generators to bid supply functions to an ISO which then sets nodal prices. Such reforms came with high hopes for efficient market outcomes at least in the presence of many generators Green [22, p. 338, 1st column], and see also Berry [5, p. 140]. This paper provides a number examples of this kind of market mechanism for two simple networks where the ISO is well-informed. It shows that even in these simple cases, in the presence of market power, loop flows and transmission constraints, SFE are not particularly efficient, intuitive or predictable. Indeed, numerous results from simpler models of electricity generation, and standard intuitions, are not supported by the illustrated SFE. This casts some doubt on whether the undertaken market reforms can be well-understood, let alone improved, without situation-specific modelling of any given electricity market.

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