MARKET INCOMPLETENESS IN REGIONAL ELECTRICITY TRANSMISSION

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Abstract

The paper analyses various proposals for the organization of regional electricity transmission in terms of the market incompleteness that they may implicitly assume. Elementary notions of variational inequalities constitute the analytical tool used throughout the paper. The discussion is conducted with reference to the flow gate debate in California and European proposals for the organization of cross border electricity trade.

1 Background

The organization of cross national transmission of electricity in Europe and the ongoing discussion of restructuring in California constitute the background of this paper. The European Commission (EC) repeatedly asserted that it will not be satisfied with 15 liberalized electricity markets (there are currently 15 nations in the European Union) but wants an integrated market. Two major bottlenecks stand in the way of that goal. One is the remaining lack of harmonization of the electricity legislation among member countries, even though plans exist at the EC level to remedy the situation (creation of a European Regulator, introduction of a new directive imposing the legal separation between transmission and other parts of the industry). The other bottleneck is the limited transmission capacity across control areas. The European Association of Transmission System Operators (ETSO) together with the Council of European Energy Regulators (CEER) and the European Commission are extensively discussing methods for dealing with access to cross border lines. It is suggested in this paper that this discussion does not take sufficient account of lessons learned in the US, especially those gained from the restructuring in California. One indeed may get the impression that Europe is (unconsciously) adopting the philosophy of decentralization which characterizes the Californian system without imposing the underlying market structure that suggested that this organization would work. If this conjecture proves true, then the difficulties encountered in California may only be a modest anticipation of those expected in Europe. This paper provides what we believe to be a potentially useful formal framework for investigating both US and European proposals for cross border electricity trade. An analysis of ETSO's proposals using the concepts developed in this paper is given in Boucher and Smeers (2001). The material of this paper consists of a formal modelling of some of the rich and extensive discussion that took place at and after the MEET meeting in Stanford in August 2000 (http://www.stanford.edu/group/EMF/meet). We accordingly focus on the dichotomy between the nodal and flow gate organizations of transmission markets and discuss various models of regional transmission organization that we analyze in terms of missing markets. Market incompleteness has indeed been identified in Wilson (1999) as a main characteristic of competitive electricity systems. The extent of market incompleteness therefore constitutes a useful yardstick to evaluate restructured electricity systems in general and the organization of regional transmission in particular.

The paper is organized as follows. Section 2 introduces some notation and the six node network used as an example throughout the paper. Even though our propositions are stated on this example, the reader can easily convince herself that they hold true in general. The section also introduces the formalism of variational inequalities and briefly recalls the principles of the nodal and flow gate organizations of transmission operations. Because the objective of the paper is to analyze regional transmission in terms of market incompleteness, we briefly recall this notion. We also distinguish, for the purpose of the analysis, between (what we somewhat arbitrarily call) physical and financial market incompleteness and missing markets. Section 3 first discusses the case where the market of transmission services is completely missing. It obviously concludes that this organization, which still pervades some proposals, results in a grossly incomplete market. The introduction of a market for transmission services whether in nodal or flow gate form is then presented. It is shown that it physically completes the market. Excessive complexity is often invoked against proposals of transmission organization that effectively deal with the intricacies of network services. As an attempt to answer some of these criticisms, Section 4 introduces the notion of an aggregate or extended flow gate. Needless to say, a concept cannot eliminate the inherent complexity of the transmission system. But its use in the organization of transmission operations may shift some of that alledged complexity from the power marketers to the Independent Systems Operators (ISO). The notion of extended flow gate introduces a dual vue of the network: power marketers procure aggregate transmission capacities to conduct their energy transactions while ISOs assemble these transmission capacities from line and ancillary services. This repackaging of the transmission services maintains a physically complete market but also allows for controlled incompleteness in case the introduction of "simplifications" is judged compulsory. The need to conduct a security constrained dispatch has been argued against the flow gate proposal. We show in Section 5 that our dual view of the market is robust with respect to this criticism. All the above discussion can be seen as taking place in the forward market. Section 6 considers both a forward and spot markets: energy and transmission transactions are concluded in the forward market and (at least partially) settled in the spot market. These two markets are both a fact of life in many restructured systems and an essential ingredient of the nodal and flow gate proposals. The introduction of a spot market explicitly calls for a representation of uncertainty and, by way of consequence, requires a more involved treatment of market completeness (financial completeness). We discuss two approaches. In the first one, transmission capacities are initially auctionned in the forward market. We show that physical market completeness can be achieved both in the nodal and flow gate models, provided the latter allows for aggregate flow gates. We next consider a financial market and show that the nodal system still achieves physical completeness but that the flow gate model requires stronger assumptions to arrive at the same result. We conclude the section by indicating that none of these approaches achieves market completeness in the sense commonly adopted in finance. Further developments of financial instruments are thus required if financial completeness is to be pursued. The main findings of the paper are summarized in the conclusion.

2 Notation, an example and some methodological reminders

2.1 The example and a primer on power flow

The discussion is conducted throughout this paper on the following example due to Chao and Peck (1998). We consider a six node network that can be seen as a two control area system (Figure 1). In this interpretation, zone I consists of nodes 1,2,3 and zone II comprises nodes 4,5,6. To focus on cross area transmission, one assumes that the lines (1–6) and (2–5) are the sole bottlenecks of the grid and the other lines have infinite capacities. In order to simplify the discussion, we suppose that there is one marketer based in each zone, purchasing and selling in both zones (e.g. marketer I purchases and sells in both I and II zones).

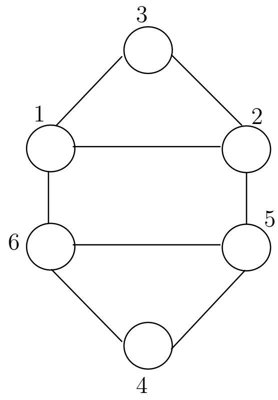

Figure 1

We assume that these marketers trade through bilateral transactions and note e_I and e_{II} the vectors of these transactions. We abuse notation by writing $e = (e_I, e_{II}) = e_I + e_{II}$, depending on the context, to refer to the vector of all transactions. In order to later introduce ISOs, we also suppose an other type of marketers that trade through nodal transactions. We let i_I, i_{II} denote the nodal injection/withdrawal of these latter marketers. We again abuse notation and write $i = (i_I, i_{II}) = i_I + i_{II}$, depending on the context.

Consumers and generators are located at the nodes of the network. A consumer is represented by his demand curve $p(q)$ (p and q are respectively the price paid by that consumer and the quantity consumed) that we assume to be monotonically decreasing. Generators are represented by their marginal cost curve $p(q)$ that we assume to be monotonically increasing. For technical simplicity we assume that $p(q)$, whether a demand or supply curve, is a continuous function.

Most of the discussion relative to the organization of transmission operations has been conducted using power distribution factors (PDF). The pros and cons of this representation of transmission are extensively discussed in

Hogan (2000b). We introduce the PDF formulation here but postpone the discussion of the extent to which we can accommodate some of these criticisms. PDFs are constructed from the DC load flow approximation of the active and reactive load flow equations. This approximation has the following properties. Assume a given topology of the network and an arbitrary swing node (taken as node 6 in Chao pand Peck (1998)). Under the DC load flow approximation, power flows obey the following properties:

(i) A unitary injection at some node k together with the corresponding withdrawal at the swing node results in a flow Γ_{mk} on the line m.

(ii) Flows are additive: the combination of injections i_k and i_ℓ from nodes k and ℓ respectively with corresponding withdrawals at the swing node results in a flow $\Gamma_{mk} i_k + \Gamma_{m\ell} i_\ell$ on line m.

The elements of Γ are the power distribution factors. Using these properties one can infer that a vector of nodal injections i results in a vector of flows Γi on the lines. It is easy to see that one can convert a vector of bilateral transactions $\tilde{e}$ into a vector $\tilde{i}$ of nodal injections and withdrawals by a matrix S ($\tilde{i} = S\tilde{e}$). Combining bilateral e and nodal i transactions we get the overall vector of nodal injections and withdrawals $Se + i$. As is frequently done in conceptual analysis of electricity transmission systems, we neglect thermal losses. It is common in academic discussions (but not sufficient for practical purposes) to only refer to thermal limits on the lines in order to represent the constraints imposed by the network on transactions. These restrict the set of feasible bilateral and nodal transactions by imposing that

$$-\overline{f} \leq \Gamma(Se + i) \leq \overline{f}.$$

For the sake of simplicity in this paper, we shall only refer in the following to the first part of these constraints namely

$$\Gamma(Se + i) \leq \overline{f} \qquad (\lambda), \tag{1}$$

As in the more standard optimization context, dual variables are associated to constraints. We let λ be this vector of dual variables (we would have written

λ^+ and λ^- in case both upper and lower bounds are imposed on the flows in the lines).

Electrical networks are subject to line outages. Their importance for the design of market mechanisms in transmission services has often been mentioned in the literature but their formal treatment remains limited. We introduce contingencies by considering a set C of network topologies, together with the power distribution factor matrices Γ^c and line capacities $\overline{f}^c$ associated to these contingencies. This leads to the additional set of network constraints and dual variables

$$\Gamma^c(Se + i) \leq \overline{f}^c \qquad (\lambda^c), \qquad c \in C. \tag{2}$$

2.2 Nodal and flow gate organizations of transmission

It is well recognized that transmission prices must include an allowance for both the fixed cost of the network and congestion charges. For the sake of simplification, we leave the discussion of the former for further work and concentrate on the latter. All references to transmission charges are thus meant to be before any allowance for the fixed costs of the network.

For the sake of this discussion, we consider that alternative designs of transmission markets are limited to the nodal and flow gate models. By definition, the nodal prices of electricity (or spot prices as they were initially introduced in Schweppe et al. (1988)) are the prices of the commodity at each node. Transmission in the nodal system is defined as a service that allows a marketer to inject at some node and withdraw at another. The marketer does not need to be concerned with the effective use of the network that this transaction requires. Transmission prices are defined as differences between nodal prices. In other words, a marketer trading between nodes 1 and 3 pays as transmission price the difference between the prices at these nodes. Transmission services can be defined as obligations or options. This distinction, although quite relevant in practice, is left out in most of this paper in order to simplify the discussion (it is briefly alluded to in Section 6.4 and in the conclusion). We therefore consider that all nodal transmission services are obligations (we shall make a similar assumption for the flow gate services defined below): a transmission service allows and requires a marketer to inject and withdraw as planned. It is the role of the ISOs to insure that the set of transactions does not violate the capacity of the network.

A flow-gate is any transmission resource deemed of limited capacity. In the original interpretation (Chao and Peck (1996)), flow-gates are physical line capacities, such as in the example, the capacities of the lines 1–6 and 2–5 in both the directions I–II and II–I respectively. The flow gate approach unbundles transmission services from node to node into uses of line capacities. Transmission services in the flow gate model allow a marketer to use a flow gate in some direction. Marketers must procure these services in amounts directly related to their energy transactions. These amounts are computed on the basis of the power distribution factors. Using the power distribution factors, a node to hub transaction is accordingly decomposed into uses of line capacities. The price of the service can thus be computed from the prices of line capacities. Some differences and similarities between the nodal and flow gate approaches can already be pointed out here by observing that dual variables λ on line constraints (1) can be interpreted as prices of line capacities while $\lambda\Gamma$ gives a vector of nodal prices. This interpretation applies to each contingency c: λ^c designates the vector of line values in contingency c while $\lambda^c\Gamma^c$ is the vector of nodal prices in that contingency. Again, transmission services in the flow gate model can be distinguished as obligations or options but we stick to the interpretation in terms of obligations: a transmission service allows and requires a marketer to use the flow gates as planned.

Flow-gates can also be defined in a more abstract way as for instance the transmission capacity between zones I and II even though it is not clear that this extension is intended in the current flow gate proposal (Chao et al (2000)). Referring to the European context, ETSO proposals (ETSO (2000)) are based on the notion of transmission capacity and hence on this extended interpretation of flow gates. This more abstract concept is thus relevant for policy discussion purposes and we accordingly refer to it as extended flow gate or transmission capacity. It is well recognized (including by ETSO itself) that transmission capacities do not have any direct physical meaning . This is not in itself a serious drawback if transmission capacities can be defined in a univoque way and adequately represent the constraints imposed by the transmission network on transactions. But these two conditions are the very issue of the debate and hence the subject of this paper.

2.3 Variational inequality formulation

Variational inequalities provide a very convenient tool to formalize alternative organizations of restructured electricity systems (Daxhelet and Smeers (2000)). This formalism is well developed in mathematical programming (Harker and Pang (1990)). It is not yet common in the restructuring literature even though the number of papers based on variational inequalities or complementarity formulations is increasing (Hobbs (2001), Kempfert (2000), Rivier et al (2000), ...). We therefore give a brief intuitive introduction to the use of variational inequalities in the context of this paper together with some assumptions that underlie all our discussion. We first introduce the following important simplifying assumption:

Assumption 1 *There is no market power and no asymmetry of information.*

The assumption implies that prices are given and known to all agents; agents cannot modify them. Also prices are equal to marginal production cost at generation nodes and to marginal utilities at consumption nodes. This also implies that the prices of electricity at the different nodes are fully determined by the set of bilateral and nodal transactions. Specifically the price at a consumption node is equal to $p(q)$ where q is the consumption at that node resulting from e and i, and $p(q)$ is the demand function at that node. A similar interpretation applies to the price at a generation node. Let e^* and i^* be vectors of bilateral and nodal transactions. Let p^* be the associated vector of nodal prices; p^*_h is the price at the hub (here node 6). A transaction $e^*_{k\ell}$ results in a settlement $(p^*_\ell - p^*_k)e^*_{k\ell}$. Changing from a transaction $e^*_{k\ell}$ to $e_{k\ell}$

implies a cost $(p^*_k - p^*_\ell)(e_{k\ell} - e^*_{k\ell})$.

Consider now the set of transactions e^*_I. Define the vector of margins $-L_I(e^*, i^*)$ (L for losses) accruing to marketer I because of these transactions, $-L_I(e^*, i^*)$ is the vector of the $(p^*_k - p^*_\ell)$ associated to the components $e^*_{k\ell}$ of e^*_I. Changing from e^*_I to e_I implies a loss $L_I(e^*, i^*)(e_I - e^*_I)$ for marketer I. A similar notation can be used to model the costs/profits accruing from nodal transactions. Changing from injection/withdrawal i^*_k to i_k at node k with equal withdrawal/injection at the hub (node 6) implies a loss $(p^*_k - p^*_h)(i_k - i^*_k)$. Introducing $-O_I(e^*, i^*)$ as the vector of nodal margins accruing to (nodal) marketer I, a change from i^*_I to i_I implies a loss $O_I(e^*, i^*)(i_I - i^*_I)$ for marketer I.

The following assumption insures the existence of a solution to almost all models appearing in the paper.

Assumption 2 *The set of nodal and bilateral transactions is compact. The mappings L_I, L_{II}, O_I, O_{II} are monotone.*

Compactness is a very mild assumption in this problem (Daxhelet and Smeers (2000)). It suffices for the rest of this paper to note that the assumption, together with standard results from variational inequality theory imply the existence of a solution to almost all models formulated in this paper (Problem XII is an exception). Monotonicity of the mappings L_τ and O_τ, $\tau = I, II$, results from the monotonicity of the supply and demand functions. It is an (important) technical assumption but it should not bother the reader.

Standard economics assumes that agents behave so as to maximize their profit or surplus or minimize their cost. Suppose first that there are no transmission constraint or charge. Then, a marketer τ would seek a set of transactions e^*_τ such that any deviation from this set would imply a loss. This can be expressed as

Problem O.1
Seek $e^*_\tau \geq 0$ such that

$$L_\tau(e^*, i^*)(e_\tau - e_{\tau *}) \geq 0 \text{ for all } e_\tau \geq 0,\ \tau = I, II. \tag{3}$$

The same can be stated for a marketer τ who trades through the hub. She would seek the set of transactions i^*_τ such that any deviation from this set would lead to a loss. This can be expressed as

Problem O.2
Seek i^*_τ such that

$$O_\tau(e^*, i^*)(i_\tau - i_{\tau^*}) \geq 0 \text{ for all } i_\tau,\ \tau = I, II. \tag{4}$$

As discussed above the network is not a copper plate and congestion restricts these behaviours. Network constraints impose limitations on transactions and hence need to be priced. The following section introduces the representation of these limitations and their impact on the above behavioural relations. Before getting into this discussion it is useful to briefly close the reminder of these introductory notions by some remarks on the incompleteness of the power market.

2.4 Incompleteness of power markets

Market incompleteness arises when the market of some good or service is missing. A market may be missing in a deterministic world when a particular service or good is not traded. This happens for instance when transmission services are not traded. Missing markets may also appear in an uncertain world when some risks are not traded, or, in other terms, when tradable assets are not sufficiently diversified to span all risks. This happens for instance when there does not exist securities to hedge the risk of a line failure. We respectively, and somewhat arbitrarily, refer to these missing markets as physical and financial and talk about physical and financial market incompleteness. Market incompleteness may make it impossible to arrive at Pareto optimality.

Even though real markets are never fully complete, this is generally not seen as a real concern. But the issue may be more serious in electricity. Wilson (1999) argues that electricity markets are inherently incomplete: electricity cannot be stored and it is currently impossible to establish a real time market that will balance price sensitive electricity supply and demand. This does not imply that consumption and generation are not equal at each moment of time. They must, for physical reason be equal, but this equality is the result of the real time action of a dispatcher and automatic generation control equipment, not of a market. This lack of instantaneous market for trading a non storable product is a first and irreducible source of market incompleteness in electricity. Because real time markets cannot be implemented with current technology, alternatives need to be created to introduce the competition intended by the restructuring process. These take the form of ex ante forward markets and/or ex post computation of prices for settlement purposes. These will never be perfect substitutes to real time markets, but they are the best we can achieve with current technology. But these alternative markets do not emerge spontaneously (Hogan (2000a)): they need to be explicitly created and failure to design them properly may further add to the inherent incompleness of the power market. The task is not trivial, as the numerous variants of restructured power systems show. Electricity is indeed a complex product consisting of a commodity (energy) bundled with various services, some of which of a continuous nature (reserve) and none of them having a value when separated from energy. These services can be unbundled and traded separately. Alternatively, they can remain bundled with energy provided the price of the bundle fully accounts for the value of these services. Market incompleteness increases if the markets of these services are not explicitly (through unbundling) or implicitly (through proper pricing in energy) created by the restructuring process. The more markets are missing, the higher the risk that the restructuring ends up in chaos. We check for market incompleteness through a simple criterion that also signals failure to reach Pareto optimality. Specifically the market will be referred to as physically incomplete whenever the analysis reveals that some products or services are valued differently by different agents in a deterministic world. The market will be financially incomplete in an uncertain world when there are not enough financial instruments to trade all risk factors so that the willingness to pay to get rid of risk may be different to different agents in some states of the world.

3 The market for transmission services

3.1 No market for transmission services

It is convenient to illustrate our approach to physical market incompleteness in restructured electricity systems by considering the case of the most grossly incomplete organization, namely one where the market for transmission services is missing. The situation is not totally unrealistic. Specifically ETSO's first proposal (ETSO (2000)) was to grant access to cross border transmission capacity on the basis of priority rules, possibly without any reference to economic valuation. This proposal was formally rejected by the European Electricity industry (EURELECTRIC) but it reappears in disguised form as a byproduct of other ETSO proposals (Boucher and Smeers (2001)). Non economic principles for allocating transmission services also underly the load relief protocol implemented by NERC in the US (Cadwalader et al (1999)). The situation can be modeled as follows.

Consider a forward energy market where marketers I and II trade bilaterally (Section 2.1). At equilibrium, marketer I seeks to optimize her position given the actions of marketer II. Similarly marketer II optimizes her position given the actions of marketer I. Limiting ourselves for the time being to a market composed of these two agents, the situation can be formalized by the following problem.

Problem I

Seek $e^* \geq 0$ s.t. $\Gamma S(e^*_I + e^*_{II}) \leq \overline{f}$ satisfying

$$L_I(e^*)(e_I - e^*_I) \geq 0 \quad \forall\ e_I \geq 0 \quad \text{s.t} \quad \Gamma S(e_I + e^*_I) \leq \overline{f}$$

and

$$L_{II}(e^*)(e_{II} - e^*_{II}) \geq 0 \quad \forall\ e_{II} \geq 0 \quad \text{s.t} \quad \Gamma S(e^*_I + e_{II}) \leq \overline{f}.$$

These relations represent the behaviours of marketers I and II respectively. It is easy to show that Problem I is equivalent to

Problem II

Seek $e^* \geq 0$ s.t. $\Gamma S(e_I^* + e_{II}^*) \leq \overline{f}$ satisfying

$$\begin{aligned} L_I(e^*)(e_I - e_I^*) + L_{II}(e^*)(e_{II} - e_{II}^*) \geq 0 \ \forall \ e \geq 0 & \\ \text{s.t } \Gamma S(e_I + e_{II}^*) \leq \overline{f} \qquad & (\lambda_I) \\ \Gamma S(e_I^* + e_{II}) \leq \overline{f} \qquad & (\lambda_{II}) \end{aligned} \tag{5}$$

This is a quasi variational inequality problem (Harker (1991)). An analysis of this model is given in Daxhelet and Smeers (2000) and Boucher and Smeers (2000)). In short, there exist multiple solutions to this problem and it is impossible to a priori ascertain which one the market will select. The notion of Social Equilibrium elaborated in Debreu (1952) provides an interpretation of that solution. This interpretation is in terms of generalized Nash equilibria (Rosen (1965)) when there is market power. In more directly interpretable terms, there exists a set of dual variables to the constraints one for each marketer (λ_I and λ_{II}), and these dual variables are not necessarily equal at equilibrium. The price of transmission services, whether defined in flow gate (λ) or nodal ($\lambda\Gamma$) terms do not necessarily equalize for the two marketers because there is no market to equalize them. At equilibrium, a marketer may thus benefit from the entirety of transmission services even if these have little value for her while the other marketer cannot access transmissions services even if she values them highly. This situation is elaborated in the following proposition.

Proposition 1 *$e^* \geq 0$ is a solution to Problem II iff there exists $\lambda_I^*, \lambda_{II}^*$ both nonnegative such that*

$$\begin{aligned} & [L_I(e^*) + \lambda_I^* \Gamma S] \, (e_I - e_I^*) \geq 0 \qquad \forall \ e_I \geq 0 \\ & [L_{II}(e^*) + \lambda_{II}^* \Gamma S] \, (e_{II} - e_{II}^*) \geq 0 \qquad \forall \ e_{II} \geq 0 \\ & \lambda_I^*[\overline{f} - \Gamma S(e_I^* + e_{II}^*)] = 0, \quad \lambda_{II}^*[\overline{f} - \Gamma S(e_I^* + e_{II}^*)] = 0 \end{aligned} \tag{6}$$

Under Assumption 2, there always exists a solution to Problem II. This solution is generally not unique.

Proof: See appendix.

It appears from this proposition that each marketer optimizes her position after respectively paying $\lambda_I^* \Gamma S e_I^*$ and $\lambda_{II} \Gamma S e_{II}^*$ for transmission services. These payments can be interpreted as flow gate prices $(\lambda_I^*, \lambda_{II}^*)$ or nodal prices $(\lambda_I^* \Gamma, \lambda_{II}^* \Gamma)$. The source of market incompleteness lies in the fact that the marginal willingness to pay for transmission services is not necessarily equal for the two marketers $(\lambda_I^* \neq \lambda_{II}^*; \lambda_I^* \Gamma \neq \lambda_{II}^* \Gamma)$ because there is no market to equalize them.

3.2 Introduce a market for transmission services

Market incompleteness in the above situation is due to the absence of a market where marketers can trade transmission services. This situation is progressively becoming of pedagogical interest though. It is now well recognized, at least in principle, that an energy market without an explicit (where transmission services are effectively defined and traded separately) or implicit (where transmission services are priced on the basis of energy prices) market for transmission services is not viable. Even though the diagnostic is becoming clear, the cure is not always well accepted. Its principle is simple: one needs to remove incompleteness by introducing a market that will equalize the marginal value of transmission services for all marketers. This is easy to do in theory, if not in practice. Mathematically, it suffices to replace the quasi variational Problem II by the following variational inequality problem.

Problem III

Seek $e^* \geq 0$ s.t. $\Gamma S(e_I^* + e_{II}^*) \leq \overline{f}$ satisfying

$$\begin{array}{c} L_I(e^*)(e_I - e_I^*) + L_{II}(e^*)(e_{II} - e_{II}^*) \geq 0 \quad \forall\, e \geq 0 \\ \text{s.t. } \Gamma S(e_I + e_{II}) \leq \overline{f}. \quad (\lambda) \end{array} \tag{7}$$

Standard variational inequality theory (Harker and Pang (1990)) can be used to show that, under Assumption 2, Problem III has a solution (see

Daxhelet and Smeers (2000)), and that, for each solution e^* to Problem III there exists λ^* such that

$$\begin{array}{l} (L_\tau(e^*) + \lambda^*\Gamma S)(e_\tau - e_\tau^*) \geq 0, \ \forall \ e_\tau \geq 0, \quad \tau = I, II \\ \lambda^* \geq 0, \qquad \overline{f} - \Gamma S(e_I^* + e_{II}^*) \geq 0, \quad \lambda^*[\overline{f} - \Gamma S(e_I^* + e_{II}^*)] = 0. \end{array} \tag{8}$$

The first condition of (8) only differs from (3) by the introduction of congestion fees $\lambda^*\Gamma S$. As we discuss below it differs from (6) by the fact that this congestion fee is unique for the two marketers.

The solution to Problem III can be interpreted in flow gate and nodal terms. Consider the flow gate first. Line utilizations are the traded services if one follows the interpretation of Chao and Peck (1996) and the flow gate model (Chao et al (2000)). There is now a market that insures that the price paid by the two marketers for the use of a unitary amount of line capacity is equal. Point to point services are the relevant services in Hogan's nodal system (Hogan (1992), Harvey et al (1997)). The equality of the price of line services implies that both marketers see the same difference of nodal prices between two locations when they trade in the nodal system. The market is thus physically complete whether in flow gate or nodal terms.

The second part of relation (8) deserves additional comments. In flow gate terms it says that the use of a line at equilibrium maximizes the revenue accruing from pricing the line capacity services at λ^* (Chao and Peck (1996)). In the nodal interpretation it says that the use of the network resulting from injections and withdrawals maximizes the value of the grid capacity when nodal injection and withdrawal rights are priced at $\lambda^*\Gamma$ (Hogan (1992)). These statements can most naturally be interpreted by introducing new agents, namely the owner(s) of the transmission system and their behavior. This is commonly done in optimization form and accompanied by a corollary usually referred to as the revenue adequacy (Hogan (1992)). These notions are restated below. Their proof in the variational inequalities context can be found in Daxhelet and Smeers (2000) (Proposition 8 in that paper).

Proposition 2
Flow gate interpretation. *Let $f^* = \Gamma S e^*$ be the flow derived from the solution e^* to Problem III. We have for each flow f_ℓ compatible with the thermal limits of the line*

$$-\lambda_\ell^*(f_\ell - f_\ell^*) \geq 0. \tag{9}$$

Nodal interpretation. *Let e^* be the solution to Problem III and $p^* = \lambda^* \Gamma$ be the nodal prices associated to λ^*. We have for all network feasible transaction vectors e*

$$-p^* S(e - e^*) \geq 0 \qquad \text{for all } e \geq 0 \text{ s.t. } \Gamma S e \leq \overline{f}. \tag{10}$$

The economic interpretation of this proposition is important. Not only does Problem III introduce a market that is absent from Problem II: it also introduces infrastructure owners who provide the services traded in that market. The interpretation of the proposition in terms of revenue adequacy is also important: it says that if the owner of an infrastructure (whether the grid or a line) collects the value of this infrastructure at the equilibrium prices and quantity, then this agent can pay for any alternative feasible use of this infrastructure.

The nodal and flow gate models are clearly equivalent under Assumptions 1 and 2. Divergences appear as soon as one comes to the design of institutions that implement these organizations. The nodal system is currently implemented in PJM and New York and reported to work well (e.g. Palmeri (2001)). In contrast, the flow gate system is not implemented but it is discussed in relation to restructuring of transmission in California. The nodal system is often criticized for its alleged complexity. A discussion of the dichotony between what appears complex ex ante (before implementation) and how complex things may turn out ex post (after implementation, taking all remedies into account) is beyond the scope of this paper. But complaints about excessive complexity even arise in smoothly running systems like PJM (e.g. Hanger (2000)). Referring to the European situation, the alleged complexity of the nodal system has also been mentioned and the need for something "simpler" advocated. ETSO accordingly proposed to design the access to cross border lines on the basis of a to be defined notion of transmission capacity. We introduced this transmission capacity as a sort of extended flow gate in Section 2.2 but did not get into details. It is to a deeper discussion of this extended flow gate/transmission capacity that we now turn.

4 Flow gate, extended flow gates and transmission capacities

4.1 Marketers want "transmission capacities"

The concept of a transmission capacity has been around for some time without an operational definition of it being proposed. Similarly ETSO builds its proposals on this notion but does not refer to any implementable definition. Our goal is to formally define a concept of transmission capacity that can be used to physically complete the market. This is done as follows. Let w^p be a vector of positive weights of line capacities (here w_{1-6}, w_{2-5}). A transmission capacity or an extended flow gate is an aggregate network constraint

$$\{e \mid w^p \Gamma e \leq w^p \overline{f}\}. \tag{11}$$

First note that a transmission capacity defined according to (11) is a relaxation of the full network constraint set (1). Second, because there are different ways to relax the constraint set (1), one can define many transmission capacities by selecting different vectors w^p. The relevant question is whether there exists a small set of transmission capacities that adequately represents the network in the sense that trading transmission services defined on the basis of these transmission capacities suffices to physically complete the market . In order to explore this question consider the following variational inequality problem.

Problem IV
Seek $e^* \geq 0$ s.t. $(w\Gamma)S(e^*_I + e^*_{II}) \leq (w\overline{f})$ satisfying

$$\begin{array}{c} L_I(e^*)(e_I - e^*_I) + L_{II}(e^*)(e_{II} - e^*_{II}) \geq 0, \ \forall \ e \geq 0 \\ \text{s.t.} \quad (w\Gamma)S(e_I + e_{II}) \leq w\overline{f}. \end{array} \tag{12}$$

Under Assumption 2, there exists a solution to this variational inequality problem and for each such solution e^* there exists at least one ξ^* such that

$$\begin{array}{l} [L_\tau(e^*) + \xi^*(w\Gamma)S]\,(e_\tau - e_\tau^*) \geq 0,\ \forall\ e_\tau \geq 0,\ \tau = I, II \\ \xi^* \geq 0, \qquad w\overline{f} - (w\Gamma)Se^* \geq 0, \qquad \xi^*[w\overline{f} - (w\Gamma)Se^*] = 0. \end{array} \tag{13}$$

It is clear that conditions (13) are equivalent to conditions (8) if $\xi^* w = \lambda^*$, $\xi^* = 1$ and one somehow insures that $\overline{f} - \Gamma Se^* \geq 0$. Leaving aside this latter condition for the time being (it is handled in the next subsection) a solution to (13) is achieved by a market that trades reservations on a newly defined transmission capacity $\lambda^*\overline{f}$, using newly defined PDF, $\Gamma^* = \lambda^*\Gamma$. At equilibrium the price of this transmission capacity is equal to 1. Assuming that $\overline{f} - \Gamma Se^* \geq 0$, the outcome of the market is equivalent to the nodal or flow gate system and hence the market is physically complete. But this outcome requires that the transmission capacity be defined appropriately, that is through weights w proportional to λ^*. In practice, this condition requires to construct the transmission capacity (the extended flow gate) at the same time as the market proceeds to find the λ^*.

A transmission capacity is thus a weighted sum of certain line capacities and the power distribution factors of this transmission capacity are constructed as aggregate PDF of the lines. A transmission capacity is thus a physical fiction. This needs not be a serious concern for power marketers who want to trade energy without being bothered by the intricacies of the electrical grid. The virtual nature of the network should not deter the ISOs either. Electrical engineers are indeed accustomed to aggregate networks or "equivalent network". The aggregation process is different but the idea of dealing with a computationally equivalent network is similar.

4.2 Who will construct the "transmission capacity"? The ISOs

The above discussion only involved power marketers operating in a forward market. It left out the question of insuring the network feasibility of the transmission services that emerge from trading transmission capacities. This latter issue, as well as the assembling of the transmission capacities themselves requires the involvment of the ISOs. Consider again the two control area system of Figure 1 and assume that the two nodal marketers introduced in Section 2.1 are assigned the role of ISOs. They can procure energy services (injections and withdrawals) at the different nodes of the grid in order to insure the network feasibility of the commercial transactions concluded by the two bilateral marketers. There are thus two ISOs, one for each zone, who can buy and sell energy in both zones in order to fulfill their mission. The interaction of these four agents is modelled by the following variational inequality problem.

Problem V

Seek (e^*, i^*), $e^* \geq 0$ s.t. $\Gamma(Se^* + i^*) \leq \overline{f}$ satisfying

$$\begin{aligned} & L_I(e^*, i^*)(e_I - e_I^*) + L_{II}(e^*, i^*)(e_{II} - e_{II}^*) + O_I(e^*, i^*)(i_I - i_I^*) \\ & + O_{II}(e^*, i*)(i_{II} - i_{II}^*) \geq 0 \\ & \text{for all } (e, i), e \geq 0 \quad \text{s.t.} \quad \Gamma(Se + i) \leq \overline{f} \end{aligned} \tag{14}$$

Applying the same argument as before, one can state the following.

Proposition 3 *Under Assumption 2, there exists a solution (e^*, i^*) to Problem V. To each solution (e^*, i^*) one can associate at least a λ^* such that*

$$\begin{aligned} & \left[L_\tau(e^*, i^*) + (\lambda^* \Gamma)S\right](e_\tau - e_\tau^*) \geq 0, \ \forall e_\tau \geq 0, && \tau = I, II \\ & \left[O_\tau(e^*, i^*) + \lambda^* \Gamma\right](i_\tau - i_\tau^*) \geq 0 && \tau = I, II \\ & \lambda^* \geq 0, \qquad \overline{f} - \Gamma(Se^* + i^*) \geq 0, \ \lambda^*[\overline{f} - \Gamma(Se^* + i^*)] = 0. \end{aligned} \tag{15}$$

Proof: See appendix.

In order to interpret relations (15), suppose that the marketers only want to trade transmission capacities without being bothered by the details of the network. The ISOs, whose job is to deal with the intricacies of the grid can be put in charge of both assembling and auctioning these transmission capacities and insuring the feasibility of the transactions of the marketers. This creates a dual view of the network (ISO/marketers) to which we will constantly refer in the following. We further characterize the respective roles of the two types of agent in the following.

4.2.1 The marketers

At equilibrium, marketers only see an aggregate transmission capacity $\lambda^*\overline{f}$. They require aggregate transmission services using the aggregate PDF, $\Gamma^* = \lambda^*\Gamma$. At equilibrium the aggregate transmission capacity has a unitary price. This is expressed by rewritting the first relation of (15) as

$$[L_\tau(e^*, i^*) + 1\Gamma^* S]\,(e_\tau - e_\tau^*) \geq 0, \quad \forall e_\tau \geq 0,\ \tau = I, II \tag{16}$$

4.2.2 The ISOs

Adding the two relations of the second line of (15) for $\tau = I$ and II, the behaviour of the ISO's is given by

$$[O_I(e^*, i^*) + \lambda^*\Gamma](i_I - i_I^*) + [O_{II}(e^*, i^*) + \lambda^*\Gamma](i_{II} - i_{II}^*) \geq 0. \tag{17}$$

We have because of the third relation of (15)

$$\begin{array}{rcl} \lambda^*\Gamma(i_I^* + i_{II}^*) & = & \lambda^*\overline{f} - \lambda^*\Gamma S(e_I^* + e_{II}^*) \\ \lambda^*\Gamma(i_I + i_{II}) & \leq & \lambda^*\overline{f} - \lambda^*\Gamma S(e_I + e_{II}). \end{array} \tag{18}$$

which leads to the following proposition.

Proposition 4 *Let (e^*, i^*) be a solution to Problem V and an associated λ^*. $(e^*, i^*), \lambda^*$ also satisfies*

$$\begin{array}{c} O_I(e^*, i^*)(i_I - i_I^*) + O_{II}(e^*, i^*)(i_{II} - i_{II}^*) \\ -\lambda^*\Gamma S[(e_I + e_{II}) - (e_I^* + e_{II}^*)] \geq 0 \\ or \\ O_I(e^*, i^*)(i_I - i_I^*) + O_{II}(e^*, i^*)(i_{II} - i_{II}^*) \\ -\Gamma^* S[(e_I + e_{II}) - (e_I^* + e_{II}^*)] \geq 0 \end{array} \tag{19}$$

for all $e_I, e_{II} \geq 0$ and all i_I, i_{II}.

Proof: See appendix.

The interpretation of this latter condition is straightforward. The ISOs acting together procure energy services i (countertrading) in order to assemble or "manufacture" a transmission capacity that they sell to the marketers. The total amount of transmission capacity sold to the marketers is $\lambda^*[\overline{f} - \Gamma(i_I^* + i_{II}^*)]$ and the PDF of this transmission capacity are given by $\Gamma^* = \lambda^*\Gamma$.

This organization is compatible with the language of ETSO even though it is not certain that it is what the European System Operators had in mind. Although this organization is not clearly articulated by the proponents of the flow-gate model (Chao et al (2000)), it also appears compatible with their writing if one accepts that flow gate are no longer physically defined and may change through time. Hogan (2000b) demonstrates that line contingencies cannot be accommodated in a pure constant physical flow gate world. It thus seems reasonable to accept an extension of the original flow gate view that requires event contingent (here through event contingent λ) flow gates. We accordingly refer to this extended interpretation of flow gates as aggregate or extended flow gates.

The above discussion is conducted under the extreme view that only a single aggregate flow gate $(\lambda^*\overline{f}, \lambda^*\Gamma)$ needs to be computed. This simplifies the notation (one does not need to introduce a set of flow gates) without restricting the generality of the argument. It should be clear (and it is quite easy to show) that any set of aggregate or pure flow gates $(w\overline{f}, w\Gamma)$ can be traded. Multiple flow gates are unnecessary from a theoretical point of view: a single transmission capacity $(\lambda^*\overline{f}, \lambda^*\Gamma)$ suffices to complete the forward transmission market. But several flow gates will be needed in practice to achieve this result in case the w are only approximations of the λ^*. We do not get into this discussion any more deeply. In order to simplify the rest of the presentation, we limit ourselves to transmission markets which trade a single aggregate flow gate.

The above interpretation requires that ISOs be allowed to play a commercial role as "assemblers" or "manufactures" of transmission capacities. The idea that ISOs can intervene in the market is common. It is an essential element of the nodal system and one of the alleged defects invoked by its opponents. Wilson (1997) recognizes that ISOs could engage in insurance activities to cover the damages caused by outages in the network. A limited commercial role of the ISOs has also been accepted by the proponents to the flow gate model. This is exemplified by Chao and Peck (1998) who argue that the ISO should procure line services in order to secure reliability. Relations (19) model a new role, independent of reliability, that gives the ISO the capability to define and modify transmission capacities in order to reflect the use and status of the network. Changing transmission capacities are a fact of life to which marketers should be prepared. In fact their demand for transmission capacities simpler to deal with than nodal prices implies that these capacities are changing. Experimenting with the construction of aggregate transmission capacities may reveal that some weights used to define aggregate flow gates are relatively constant through time. This would make it possible to define approximate transmission capacities that can be announced well in advance. Needless to say divergence between ex ante announced and real time transmission capacities will always exist and this will require a resolution in real time. It may also be possible that sufficiently robust weights cannot be found because the real time value of line capacities is changing too dramatically. The current debate among proponents of the flow gate and nodal models shows that the issue is largely unresolved. What is clear however, is that any proposal for using fixed aggregate capacities needs to be accompanied by remedies to accommodate these divergences between ex ante and real time transmission capacities. The proponents of the flow gate model recommend to go to the spot market to resolve this divergence. We turn to this question in Section 6 after first extending the current discussion to security constrained dispatch.

5 The security constrained dispatch

The difficulty of a priori defining a few relevant, commercially important, and constant through time flow gates has been pointed out by Hogan (2000b) and Ruff (2000). We here turn to one of their major arguments, namely that the need for a security constrained dispatch makes it impossible to limit oneself to a few commercially significant flow-gates, let alone to identify them in advance. The argument goes as follows: for security reasons, ISOs restrict energy transactions to those that remain feasible under different contingencies. This is usually done by only accepting transactions that satisfy constraints representing these different contingencies. The importance of the argument has been pointed out numerically in Boucher et al (1998). We take up the issue of security constrained dispatch and successively consider two cases depending on whether corrective actions after an incident are allowed or not.

5.1 Version 1: No endogenous corrective action

Consider first the case without corrective actions: transactions accepted by the ISOs must remain feasible for the network in case each one of a set of contingencies occurs. The question addressed in this section is whether the notion of aggregate transmission capacities and their construction by ISOs that is, the dual ISO/marketer view of the network, carries through to this more complex situation. In order to explore the issue consider the following variational inequality problem

Problem VI

Let $c \in C$ be the states of the network associated to the relevant contingencies. Seek $(e^*, i^*), e^* \geq 0$ s.t. $\Gamma^c(Se^* + i^*) \leq \overline{f}^c, c \in C$, satisfying

$$\begin{aligned}&L_I(e^*, i^*)(e_I - e_I^*) + L_{II}(e^*, i^*)(e_{II} - e_{II}^*) + O_I(e^*, i^*)(i_I - i_{II}^*)\\&+O_{II}(e^*, i^*)(i_{II} - i_{II}^*) \geq 0\end{aligned}$$

for all (e, i), $e \geq 0$ satisfying $\Gamma^c(Se + i) \leq \overline{f}^c$, $c \in C$.

The following proposition extends previous statements of Section 4.

Proposition 5 *Under Assumption 2, there exists at least one solution* (e^*, i^*) *to Problem* VI*. To each solution* (e^*, i^*) *one can associate at least one vector of dual variables* λ^{c*}*,* $c \in C$*, such that*

$$\begin{aligned}&(L_\tau(e^*, i^*) + \textstyle\sum_c \lambda^{c*}\Gamma^c S)(e_\tau - e_\tau^*) \geq 0,\ \forall e_\tau \geq 0, \tau = I, II\\&(O_\tau(e^*, i^*) + \textstyle\sum_c \lambda^{c*}\Gamma^c)(i_\tau - i_\tau^*) \geq 0,\ \forall i_\tau,\ \tau = I, II\\&\lambda^{c*} \geq 0,\ \overline{f}^c - \Gamma^c(Se^* + i^*) \geq 0,\ \lambda^{c*}[\overline{f}^c - \Gamma^c(Se^* + i^*)] = 0, c \in C.\end{aligned} \tag{20}$$

Proof: See appendix.

Proposition 5 can be directly interpreted in the dual ISO/marketer context. ISOs deal with nodal injections/withdrawals and contingent line capacities and PDFs. They assemble them to construct and auction the firm extended flow gate in amount $\sum_c \lambda^{c*}(\overline{f}^c - \Gamma^c i^*)$. Marketers only see and trade the firm aggregate flow gate. The price of the extended flow gate is equal for all agents in the market which is therefore physically complete. In order to complete their mission ISOs trade in the energy and transmission markets.

5.2 Version 2: With endogenous corrective actions

We now turn to the case where corrective actions are possible after the incident. This implies that the ISOs have managed to procure optional resources allowing them some recourse after an incident. These services are physical options traded on an ancillary service market. They are not our concern here where we simply assume that the markets that price these physical options exist (see Daxhelet and Smeers (2001) for a modeling of these markets). The set of acceptable transactions is thus larger than the domain of those transactions that remain feasible under the listed set of incidents without corrective action. The situation is modeled through the following variational inequality problem.

Problem VII

Seek $(e^*, (i^{c*}, c \in C))$, $e^* \geq 0$, s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c, c \in C$ satisfying

$$L_I(e^*, i^*)(e_I - e_I^*) + L_{II}(e^*, i^*)(e_{II} - e_{II}^*) + \textstyle\sum_c [O_I(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}(e^*, i^{c*})(i_{II}^c - i_{II}^{c*})] \geq 0 \tag{21}$$

for all $(e, (i^c, c \in C))$, $e \geq 0$ s.t $\Gamma^c(Se + i^c) \leq \overline{f}^c, c \in C$.

This problem satisfies the following proposition:

Proposition 6 *Under Assumption 2, there exists at least one solution* (e^*, i^*) *where* $i^* = (i^{c*}, c \in C)$ *to Problem* VII. *To each solution* (e^*, i^*) *one can* *associate at least one vector of dual variables* λ^{c*}, $c \in C$, *such that*

$$\begin{array}{l}(L_\tau(e^*, i^*) + \sum_c \lambda^{c*}\Gamma^c S)(e_\tau - e_\tau^*) \geq 0,\ \forall e_\tau \geq 0, \tau = I, II \\ (O_\tau(e^*, i^{c*}) + \sum_c \lambda^{c*}\Gamma^c)(i_\tau^c - i_\tau^{c*}) \geq 0,\ \forall i_\tau,\ \tau = I, II, c \in C \\ \lambda^{c*} \geq 0,\ \overline{f^c} - \Gamma^c(Se^* + i^{e*}) \geq 0, \lambda^{c*}[\overline{f}^c - \Gamma^c(Se^* + i^{c*})] = 0, c \in C\end{array} \tag{22}$$

Proof: See appendix.

The dual ISO/marketer view of the network applies as before. It again insures market completeness through the auctioning of a single aggregate flow gate in amount $\sum_c \lambda^{c*}(\overline{f}^c - \Gamma^c i^{c*})$.

5.3 Comments

Whether we deal with a priori exogenously specified or ex post endogenously determined corrective actions, the behaviour of the marketers (first relation of (20) and first relation of (22)) can be written as

$$[L_\tau(e^*, i^*) + 1\Gamma^* S]\,(e_\tau - e_\tau^*) \geq 0, \quad \forall e_\tau \geq 0,\ \tau = I, II \tag{23}$$

where $\Gamma^* = \sum_c \lambda^{c*}\Gamma^c$. This extension of (16) expresses that marketers only see a single aggregate transmission capacity. The price of this aggregate flow gate is equal to 1 for both the marketers and the ISOs.

To discuss the role of the ISOs, consider the more complex case with recourse actions. The behaviour of the ISOs is represented by

$$[O_I(e^*, i^{c*}) + \lambda^{c*}\Gamma^c](i_I^c - i_I^{c*}) + [O_{II}(e^*, i^{c*}) + \lambda^{c*}\Gamma^c](i_{II}^c - i_{II}^{c*}) \geq 0, c \in C \tag{24}$$

This means that the ISOs anticipate their corrective actions for each $c \in C$. We have, because of the third relation of (22),

$$\begin{array}{rcl}\lambda^{c*}\Gamma^c(i_I^{c*} + i_{II}^{c*}) & = & \lambda^{c*}\overline{f}^c - \lambda^{c*}\Gamma^c S(e_I^* + e_{II}^*) \\ \lambda^{c*}\Gamma^c(i_I^c + i_{II}^c) & \leq & \lambda^{c*}\overline{f} - \lambda^{c*}\Gamma^c S(e_I + e_{II})\end{array} \tag{25}$$

which leads to the following proposition.

Proposition 7 *Let (e^*, i^*) and $\lambda^* \geq 0$ be a solution to Problem V together with an associated λ^*. (e^*, i^*) also satisfies*

$$\begin{array}{c} \sum_c O_I(e^*,i^*)(i_I^c - i_I^{c*}) + O_{II}(e^*,i^*)(i_{II}^c - i_{II}^{c*}) \\ - \sum_c \lambda^{c*}\Gamma^c S[(e_I + e_{II}) - (e_I^* + e_{II}^*)] \geq 0 \\ or \\ \sum_c O_I(e^*,i^*)(i_I^c - i_I^{c*}) + O_{II}(e^*,i^*)(i_{II}^c - i_{II}^{c*}) \\ -\Gamma^* S[(e_I + e_{II}) - (e_I^* + e_{II}^*)] \geq 0 \end{array} \quad (26)$$

for all $e_I, e_{II} \geq 0$.

Proof: The proof follows from the above discussion.

This proposition extends the interpretation of the role of the ISOs already given in Proposition 4. ISOs acting together procure optional services i^c (contingent countertrading) in order to assemble or "manufacture" a transmission capacity that they auction to the marketers. The existence of these optional services enlarge the set of feasible firm transmission capacity. The total amount of transmission capacity auctionned to marketers is $\sum_c \lambda^{c*}(\overline{f}^c - \Gamma^c i^{c*})$ and the PDF of this transmission capacity is $\Gamma^* = \sum \lambda^{c*}\Gamma^c$.

The above discussion only shows the existence of aggregate flow gates or transmission capacities. It does not provide an algorithm for constructing them. Recall that a single aggregate flow gate will suffice in theory. In practice we need more flow gates if these are to be defined and auctionned well in advance. In principle, the more (extended) flow gates, the better for market completeness. The theoretical importance of introducing at least one aggregate flow gate is obvious. Without aggregate flow gates, market completeness requires that all contingent line capacities be traded. With aggregate flow gates, it suffices that a sufficient large set of a priori defined non physical flow gates be traded.

This discussion somewhat delineates the value of the argument that security constraint dispatch is incompatible with the flow gate approach. The restrictions imposed by the security constrained dispatch can indeed be accommodated in a dual ISO/marketer view provided the ISOs properly assemble aggregate transmission capacities that are then traded on the market. Needless to say this dual view only makes practical sense if the aggregate transmission capacities are sufficiently stable through time. The problem is that different views are held about this stability as shown by the arguments expressed in the Oren/Ruff debate (http://www.stanford.edu/group/EMF/meet). The issue has not been systematically researched and hence remains largely open at this stage. An answer would certainly constitute an important piece of information in the debate between proponents of the flow gate and nodal models in the US. But this piece of information is vital for the European system that is supposed to be entirely based on the notion of transmission capacities. There seems to be no alternative to numerically testing the stability of the transmission capacities in different contexts and under different assumptions.

6 Settlement of transmission services on the spot market

The above discussion is conducted with respect to the forward energy and transmission markets. Specifically we only looked at completeness of the forward market. The ever changing nature of electricity renders deviations from transactions concluded in the forward market unavoidable. Demand will not be as expected and generators and lines may default with the result that ISOs will have to reconfigure their system. Because of these uncertainties both the nodal and flow gate models foresee that transmission services be eventually settled in the spot market. We disregard here any distinction between real time spot and ex post computed prices and assimilate the latter to the former. We accordingly work under the assumption that energy and transmission transactions are concluded in the forward market and settled in the spot market in both the nodal and flow gate models. Our task in this section is to extend the dual ISO/marketers view of the network to this duality of markets. We do this by considering the two-stage framework depicted in Figure 2.

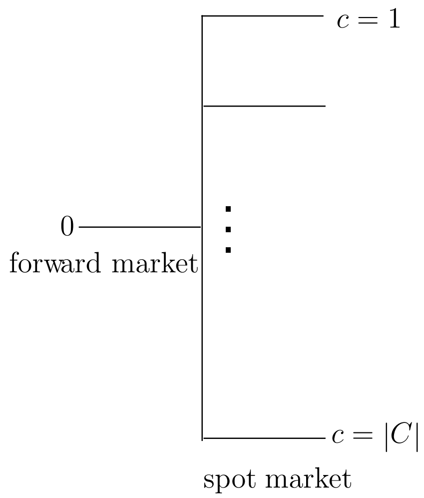

Figure 2 The two stage framework

In this framework, energy and transmission contracts are concluded in the first stage (stage 0) which represents the forward market. They are then settled in each state of the world $c \in C$ of the second stage (stage 1) after information on contingencies has been revealed. This represents the spot market. This two-stage framework can be traced to both the fields of stochastic programming in mathematical programming (e.g. Birge and Louveaux (1997)) and to introductions to finance theory (e.g. chapter 1 of Duffie (1996)). This section combines the two lines of reasoning.

The standard interpretation of the two stage model in finance is that investors assemble a portfolio of tradable assets in stage 0 while the payoffs from this portfolio accrue in stage 1. In the particular context of transmission across control areas, marketers acquire physical or financial transmission rights in the forward market. Speculators only acquire financial rights. In real time, ISOs take over all transmission resources and operate the system so as to meet the constraints of the network. In order to do so, they may exert physical options procured before. As a by-product of their action, they also compute ex post electricity spot prices that are used to settle transmission services and deviations from energy transactions. This organization can be formalized as follows.

Assume that the behaviour of a marketer τ is driven by a utility function $U_\tau(x^0_\tau, (x^c_\tau, c \in C))$ where x^c_τ is the payoff collected by the marketer τ in state of the world c and x^0_τ is the payoff collected at stage 0. U is not necessarily an expected utility function (in fact there is no probability in the statement of the problem); it suffices that it is strictly concave and includes the argument x^c_τ for each state of the world c. Let α^c_τ be the marginal utility of marketer τ in state of the world c (by assumption $\alpha^c_\tau > 0$). We assume without loss of generality and in order to simplify the discussion that the marginal utility α^0_τ of marketer τ in stage 0 is equal to one. Because energy and transmission prices are entirely determined by the e and i^c in this model, α^c_τ can always be expressed as a function of these variables only. Let $L^c_\tau(e, i^c)$ be, as before, the marginal opportunity cost due to energy settlement of marketer τ in state c ($L^c_\tau(e, i^c)$ may be independent of c in case the energy transaction is perfectly hedged). Were it not for transmission the behaviour of marketer τ would be driven by her marginal utility

$$\sum_{c \in C} \alpha^c_\tau(e, i^c) L^c_\tau(e, i^c) \tag{27}$$

Transmission is priced on the spot market: let λ^{c*} be the vector of spot values of lines used in state c. The marginal utility of trader τ after paying for both energy and transmission is equal to

$$\sum_{c \in C} \alpha^c_\tau(e, i^c)(L^c_\tau(e, i) + \lambda^c \Gamma^c S) \tag{28}$$

Given this background we now successively model the spot and forward markets.

6.1 The spot market (second stage of the equilibrium)

Consider the following set of problems (one problem for each $c \in C$).

Problem VIII^{2,c}
Given e^*, seek i^{c*} s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$ satisfying

$$O_I^c(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^c(e^*, i^{c*})(i_{II}^c - i_{II}^{c*}) \geq 0 \tag{29}$$

for all i^c s.t. $\Gamma^c(Se^* + i^c) \leq \overline{f}^c$.

As already argued several times before, Assumption 2 guarantees that each Problem $VIII^{2,c}$ has a solution i^{c*}. Moreover, for each such solution i^{c*}, there exists at least one λ^{c*} that satisfies

$$\lambda^{c*} \geq 0, \qquad \lambda^{c*}[\overline{f}^c - \Gamma^c(Se^* + i^{c*})] = 0. \tag{30}$$

Problem $VIII^{2,c}$ assumes that the injections/withdrawals i^{c*} of the ISOs clear the market in real time. This assumption is far from trivial but it is directly inspired by current movements in the market. Efforts are currently underway on the East Coast to arrive at a single spot market operated by PJM, NEPOOL and NYPP (Stavros (2000)). This type of organization is only possible if there exists a balancing market accross the different control areas. It assumes that an equilibrium is reached on this balancing market. Procedures for attaining this equilibrium across different systems in interaction are discussed in Cadwalader et al. (1999). The following proposition may help liaise this equilibrim interpretation to the more ad hoc computation of ex post prices that is effectively implemented as a substitute to real time market. We assimilate spot and balancing markets in the following.

Proposition 8 *Let i^{c*} be the solution to Problem $VIII^{2,c}$ in state of the world c. i^{c*} is also the solution of a full re-optimization of the dispatch by all ISOs operating jointly.*

Proof: See appendix.

This well known result, here applied to the sole spot market, says that this joint optimization simulates a balancing market in equilibrium. It also justifies a joint optimization of the system by all ISOs in order to both efficiently manage differences between the forward and real time market and produce spot prices that can be used to settle those differences. One may wonder about the outcome of the spot market when the joint equilibrium cannot be implemented. In order to explore this situation, consider the case where each ISO tries to resolve the congestion problem through his own means. In order to do so we introduce the set of quasi variational inequality problems.

Problem IX^{2,c}
Seek i^{c*} s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$ satisfying

$$O_I^c(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^c(e^*, i^{c*})(i_{II}^c - i_{II}^{c*}) \geq 0$$

for all i_I^c, i_{II}^c s.t.

$$\begin{aligned} &\Gamma^c(Se^* + i_I^{c*} + i_{II}^c) \leq \overline{f}^c \\ &\Gamma^c(Se^* + i_I^c + i_{II}^{c*}) \leq \overline{f}^c. \end{aligned} \tag{31}$$

Reproducing the reasoning of Section 3.1 one can show that there exists a solution to this problem. Moreover to each such solution one can associate multipliers λ_I^{c*}, $\lambda_{II}^{c*} \geq 0$ such that

$$\begin{aligned} &\lambda_I^{c*}[\overline{f}^c - \Gamma^c(Se^* + i_I^{c*} + i_{II}^{c*})] = 0 \\ &\lambda_{II}^{c*}[\overline{f}^c - \Gamma^c(Se^* + i^{c*} + i_{II}^{c*})] = 0. \end{aligned} \tag{32}$$

We can state the following proposition.

Proposition 9 *Let i^{c*}, $c \in C$ be a solution to Problem $IX^{2,c}$. It is also a solution to a pair of optimization problems where each ISO tries to relieve congestion at its own minimal cost, given the actions of the other ISO. This solution does not correspond to a full re-optimization of the dispatch nor to an equilibrium of the spot market. There are many such solutions.*

Proof: See appendix.

Problem $IX^{2,c}$ and Proposition 9 are stated with reference to the six nodes/two zones examples. In this example, refraining from global optimization by the ISOs amounts to resorting to a pair of single ISO actions. It should be clear that Proposition 9 can be extended to more general situations than the two control areas allowed by the six node example. It can indeed be adapted to a multi (> 2) control area context where single, bilateral or multilateral actions by ISOs take place. A particular relevant example is the one of a bilateral action by two ISOs controlling the two sides of a congestion in a multi (> 2) control area system. What this extended proposition would say is that anything that is not a joint optimization will not be an equilibrium of the spot market. Moreover, there will be many solutions of this type that are not equilibria of the spot market.

The non unicity of the solution to Problem $IX^{2,c}$ and its associated non optimality may be claimed to be irrelevant in practice. It is indeed often argued that the re-optimization of the dispatch involved in nodal prices systems only results in small costs and hence is not important. This view is misleading: the cost of an optimally operated system may be small, while the cost of not operating it optimally may be quite high. This is especially true when this non optimal operation is associated with perserve incentives to game the system. Besides this non optimality, a key issue for the design of the market is that the partial reoptimization approach does not lead to a single set of spot prices and hence makes it impossible to settle transmission services through spot prices. Specifically, for each particular solution to Problem $IX^{2,c}$, each ISO (or group of ISOs in a more general context) will produce different spot prices. This indicates that the market is physically incomplete whenever ISOs only engage in non global actions. The only possible outcome in reaction to this absence of spot prices is to socialize costs or to allocate them to agents that are arbitrarily defined as responsible for them. These approaches are bound to result in economically unjustified allocation of re-dispatching costs and possibly in perverse incentives to game the system.

A minimal condition for achieving a (possibly imperfect) equilibrium on the spot market is that this latter exists. It is remarkable that the existence of a balancing and hence of a spot market is not mentioned in ETSO proposals (the European Electric Industry pointed to this very serious flaw). In the absence of this spot or balancing market, ETSO can only recommend cooperation between ISOs to relieve congestion. Full cooperation is a possibility but ETSO also accepts partial reoptimization by subgroups of ISOs. This combines all defects: there is no spot price to settle transmission transactions, and no guarantee of redispatch optimality. This will result in high costs and perverse incentives.

6.2 The forward market (first stage of the equilibrium)

Using the marginal utilities defined in the introduction to this section, we assume that marketers select their portfolio of transmission rights in the forward market so as to satisfy the following variational inequality:

Given expectations on the λ^{c*}, $c \in C$, marketer τ seeks $e^*_\tau \geq 0$ s.t.

$$\left\{\sum_c \alpha^c_\tau [L^c_\tau(e^*, i^{c*}) + \lambda^{c*}\Gamma^c S]\right\} (e_\tau - e^*_\tau) \geq 0 \text{ for all } e_\tau \geq 0. \quad (33)$$

These conditions can easily be rewritten as

$$\left\{\sum_c [\alpha^c_\tau L^c_\tau(e^*, i^{c*}) + \lambda'^{c*}_\tau \Gamma^c S]\right\} (e_\tau - e^*_\tau) \geq 0, \text{ for all } e_\tau \geq 0,$$

where $\lambda'^{c*}_\tau = \alpha^c_\tau \lambda^{c*}$ and hence

$$\lambda'^{c*}_\tau \geq 0, \qquad \overline{f}^c - \Gamma^c(Se^* + i^{e*}) \geq 0, \qquad \lambda'^{c*}_\tau[\overline{f}^c - \Gamma^c(Se^* + i^{c*})] = 0. \quad (34)$$

Except for the case where $\alpha^c_I = \alpha^c_{II}$, $\sum_c \lambda'^{c*}_I \Gamma^c$ is not equal to $\sum_c \lambda'^{c*}_{II} \Gamma^c$. The market is therefore physically incomplete. Something additional is needed to complete it. We consider the two solutions most commonly proposed in the literature namely to auction transmission capacities in the forward market (physical rights) or to introduce a financial market for these rights.

6.3 Completing the market

6.3.1 Auction transmission capacities

The auctioning of transmission capacities, whether defined in nodal or flow gate terms, is either proposed or already in existence in most restructured systems. In the US, PJM and NY auction financial transmission rights. Attempts to auction transmission capacities are also beginning in Europe. As before, we assume that ISOs assemble the transmission capacities or extended flow gates and auction them. As argued before, this assumption is compatible with the flow gate proposal (Chao and Peck (1996), Chao et al (2000)) provided one accepts that flow gates do not necessarily correspond to fixed physical capacities and are in fact extended flow gates. We model the situation through the following variational principle that we interpret later.

6.3.1.1. Mathematical formulation

Problem X
Seek $(e^*, (i^{c*}, c \in C))$, $e^* \geq 0$ s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$ satisfying

$$\left[\sum_c \alpha_I^c L_I^c(e^*, i^{c*})\right](e_I - e_I^*) + \left[\sum_c \alpha_{II}^c L_{II}^c(e^*, i^{c*})\right](e_{II} - e_{II}^*) \\ + \sum_c \left[O_I^c(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^c(e^*, i^{c*})(i_{II}^c - i_{II}^{c*})\right] \geq 0 \quad (35)$$

for all $(e, (i^c, c \in C)), e \geq 0$ s.t. $\Gamma^c(Se + i^c) \leq \overline{f}^c,\ c \in C$.

Assumption 2 again guarantees the existence of a solution $(e^*, (i^{c*}, c \in C))$ to Problem X. Moreover to each solution, one can associate at least one vector of $\lambda^{c*} \geq 0, c \in C$ such that

$$\begin{array}{l} \left[\sum_c \alpha_\tau^c L_\tau^c(e^*, i^{*c}) + \sum_c \lambda^{c*}\Gamma^c S\right](e_\tau - e_\tau^*) \geq 0, \text{ for all } e_\tau \geq 0,\ \tau = I, II \\ \left[O_\tau^c(e^*, i_\tau^*) + \lambda^{c*}\Gamma^c\right](i_\tau^c - i_\tau^{c*}) \geq 0 \text{ for all } i_\tau^c,\ c \in C,\ \tau = I, II \\ \overline{f}^c - \Gamma^c(Se^* + i^*) \geq 0, \quad \lambda^{c*}\left[\overline{f}^c - \Gamma^c(Se^* + i^*)\right] = 0, \end{array} \quad (36)$$

The first line of (36) can be interpreted as follows. Marketers pay $\sum_c \lambda^{c*}\Gamma^* Se^*$ at stage 0 for transmission services. This payment is accounted for with a unitary marginal utility (recall that we assumed $\alpha_\tau^0 = 1$). Marketers then incur an additional payment $L_\tau^c(e^*, i^{c*})e^*$ when energy transactions are settled. $\sum_c \lambda^{c*}\Gamma^c S$ can thus be interpreted as the vector of present transmission prices (at stage 0) on the forward market. Because both marketers see the same price of transmission capacities in this forward market, the market is physically complete.

The role of the ISOs in the forward market can be described on the basis of the second line of (36). The i^{c*} of the solution $(e^*, (i^{c*}, c \in C))$ to Problem X satisfies

$$\sum_c \sum_\tau \left[O_\tau^c(e^*, i_\tau^*) + \lambda^{c*}\Gamma^c\right](i_\tau^c - i_\tau^{c*}) \geq 0, \text{ for all } i_\tau^c \tag{37}$$

Noting from the third line of (36) that

$$\lambda^{c*}\Gamma^c i_\tau^{c*} = \lambda^{c*}\overline{f}^c - \lambda^{c*}\Gamma^c S e^*, c \in C$$

and that from (35)

$$\lambda^{c*}\Gamma^c i_\tau^c \leq \lambda^{c*}\overline{f}^c - \lambda^{c*}\Gamma^c S e, c \in C$$

we obtain the following variational inequality

$$\sum_c \sum_\tau O_\tau^c(e^*, i^{c*})(i_\tau^c - i_\tau^{c*}) - (\sum_c \lambda^{c*}\Gamma^c S)(e - e^*) \geq 0, \ \forall\, e \geq 0, \ \forall i_\tau^c, c \in C. \tag{38}$$

This says that the ISOs maximize the profit accruing from selling transmission rights e^* and buying the energy services i^{c*} in each state of the world c in order to assemble these rights.

6.3.1.2. Discussion

It remains to cast the auctionned transmission services both in nodal and flow gate terms. The solution to Problem X is directly interpretable in nodal point to point services. ISOs sell bilateral transmission services e^* that they charge at nodal forward price $(\sum_c \lambda^{c*}\Gamma^c S)$ differences. For this they buy nodal injection/withdrawal services i^{c*} that they pay at nodal spot prices. Consider now the flow gate version of the model. One can think of at least two interpretations. One is to auction lines capacities contingent on the state of the world. This would result in contingent prices λ^{c*}. As argued by both the proponents and opponents of the flow gate model, this is too complex. The second interpretation is to auction non physical flow gate constructed through an aggregation process. This is expressed by relation (38) in which ISOs "assemble" a non contingent extended flow gate in amount $(\sum_c \lambda^{c*}\Gamma^c S)e^*$ by buying contingent line and generation services (these latter are not represented in this model). They then auction the extended flow gate capacity to marketers. This interpretation complies with the dual ISO/marketer view of the network: (i) marketers see aggregate flow gates (e.g. one flow gate of capacity $\sum_c \lambda^{c*}\Gamma^c S e^*$ of unitary price that they request according to aggregate PDF $\sum_c \lambda^{c*}\Gamma^c$) (ii) system operators see line capacities in different contingencies and take care of these contingencies to produce a firm aggregate flow gate. As before, we adopted the extreme view that one only needs a single aggregate flow gate. This is notationally convenient but purely conceptual. The practical issue is whether a small number of firm aggregate flow gates or transmission capacities can be reasonably defined in advance and remain sufficiently stable.

The nodal and flow gate interpretations both assume that one auctions transmission capacities that remain feasible for all contingencies envisaged by the ISOs. This is akin to Hogan (2000b) and Ruff (2000) argument that nodal prices are determined in a security constrained dispatch (Section 5) and that flow gates should also be defined to account for these security constraints. This is also related to the issue of revenue adequacy that is extensively discussed in Hogan (2000b). Note that the constraints of (35) guarantee that any transmission service procured in the forward market remains feasible in the spot market. This insures revenue adequacy: the congestion charges collected in the spot market are higher than the payment at spot prices of any other bundle of feasible transmission contracts inherited from the forward market. In contrast, a revenue defficiency may arise when the allocation of node to node services inherited from the forward auction is not feasible in real time. This defficiency does not arise in this model because all real time constraints are anticipated by the ISOs in the forward market. The solution to Problem X therefore allows one to generalize the revenue adequacy property of Hogan

(1992). But this generalization only holds because one assumes that the ISOs envisage the whole set of possible contingencies in the forward market. This assumption is standard in finance models where one supposes that all agents foresee the same set of states of the world, possibly with different estimates of their likelihood. The assumption may be heroic here. If it is not satisfied, the ISOs may found themselves managing real time contingencies different from those envisaged in the forward market. Revenue adequacy may then fail to be satisfied (see Hogan (2000b)).

As a last remark we note that the above interpretation is in terms of physical contracts, whether for the nodal or flow gate model. The bilateral contracts e^* negotiated by the marketers, for which they have obtained transmission rights in the auction can indeed be completed in real time. We now turn to an other version of the model where financial products are introduced as an alternative tool for managing transmission services.

6.3.2 Introduce a financial market

The nodal and flow gate models expose marketers to the same transmission risk, namely contingent differences of nodal prices. An extensively discussed issue is whether both systems can offer the same hedges against this risk. Hedging instruments take the form of financial derivatives (forward, options of various types). We hereby introduce forward nodal and flow gate contracts but leave out options contracts for the sake of simplicity.

6.3.2.1. Mathematical formulation

In order to define nodal and flow gate contracts, consider an arbitrary state of the network represented by the matrix of power distribution factors Γ^0. For any nonnegative vector λ^0 of line values, $\lambda^0\Gamma^0$ is a vector of nodal prices. To each node we associate a contract, that gives as payoff in state of the world c the component of $\lambda^{c*}\Gamma^c$ associated to that node. This contract requires an upfront payment equal to the corresponding component of $\lambda^0\Gamma^0$ in stage 0. A portfolio of these contracts is noted by θ. As is common in finance models we also assume a risk free asset. In order to simplify the presentation, and because of the short horizon covered by this two stage model, we suppose a zero interest rate. The risk free asset thus has a unitary value both in stage 0 and in all states of the world c at stage 1. Because there is a financial market, we permit speculators who are not involved in energy or transmission services to also trade these nodal contracts.

Financial contracts introduce a new good in the economy which now trades energy, transmission services and financial transmission contracts. This requires an adaptation of our representation of marketers. These were modeled by a utility function $U_\tau(x^0_\tau, (x^c_\tau, c \in C))$ where x^c_τ is the payoff from the settlement of energy and transmission transactions in state c. They will now be represented by the utility function $U_\tau((x^0_\tau + y^0_\tau), ((x^c_\tau + y^c_\tau), c \in C))$ where y^0_τ and y^c_τ are the revenue accruing from financial contracts. The marginal utilities are still noted α^c_τ but now depend on e, i and y that is $\alpha^c_\tau(e, i, (y^c_\tau, c \in C))$. Speculators are also represented by a utility function $V(z^0, (z^c, c \in C))$ where z^0 and z^c respectively denote the revenue accruing to speculators from their trading of financial transmission contracts. We note δ the portfolio of the contracts held by a speculator and $\beta^c(e, i, z^0, (z^c, c \in C))$ his marginal utility in state of world c ($\beta^0(e, i, z^0, (z^c, c \in C)) = 1$). For the sake of simplicity in the presentation, we only assume a single speculator. Given this extended set up, a marketer's position in energy and transmission contracts is a solution to the following variational inequality problem:

Problem XI

Seek $e^*_\tau \geq 0$, θ^* satisfying

$$\begin{aligned}&\textstyle\sum_c \alpha^c_\tau \left[L^c_\tau(e^*, i^{c*}) + \lambda^{c*}\Gamma^c S\right](e_\tau - e^*_\tau)\\ &\quad + \left(\lambda^{0*}\Gamma^0 - \textstyle\sum_c \alpha^c_\tau \lambda^{c*}\Gamma^c\right)(\theta_\tau - \theta^*_\tau) \geq 0\\ &\quad \text{for all } e_\tau \geq 0, \theta_\tau, \tau = I, II.\end{aligned} \tag{39}$$

The speculator's position is a solution to

Problem XIbis
Seek δ^* satisfying

$$(\lambda^{0*}\Gamma^0 - \sum_c \beta^c \lambda^{c*}\Gamma^c)(\delta - \delta^*) \geq 0 \text{ for all } \delta.$$

The term $(\lambda^{0*}\Gamma^0 - \sum_c \alpha_\tau^c \lambda^{c*}\Gamma^c)$ appearing in (39) is the contribution to the utility of marketer τ accruing from her trading financial transmission contracts. It can be explained as follows: $\lambda^{0*}\Gamma^0\theta_\tau^*$ is the upfront payment for the portfolio of contracts θ_τ^*, and is accounted for with unitary marginal utility. $\lambda^{c*}\Gamma^c\theta_\tau^*$ is the payment accruing from this portfolio of contracts in state of the world c. It is accounted for with marginal utility α_τ^c. Together, these additional terms express that marketer τ takes a position θ_τ^* in financial contracts such that any deviation θ_τ from θ_τ^* can only deteriorate her position. A similar interpretation holds for the speculator.

Finance models commonly rely on a no arbitrage assumption. In this two stage model, the no arbitrage assumption states that it is impossible at equilibrium to find a portfolio of nodal contracts θ that brings a positive profit in certain states c of stage 1 without investing a positive amount in stage 0. In other words, there does not exist any portfolio θ such that $\lambda^{0*}\Gamma^0\theta \leq 0$ and $\lambda^{c*}\Gamma^c\theta \geq 0$ for all c with $\lambda^{c*}\Gamma^c\theta > 0$ for at least one c. The no arbitrage assumption is at the origin of valuation methods based on so called "risk neutral probability". These notions are used in the following lemma.

Lemma 1 *A solution to Problem XI satisfies*

$$\lambda^{0*}\Gamma^0 = \sum_c \alpha_\tau^c \lambda^{c*}\Gamma^c; \lambda^{0*}\Gamma^0 = \sum_c \beta^c \lambda^{c*}\Gamma^c. \tag{40}$$

Moreover, under the no arbitrage assumption, there exists at least one "risk neutral probability" such that

$$\lambda^{0*}\Gamma^0 = \sum_{c\in C} \pi^c(\lambda^{c*}\Gamma^c); \pi^c > 0,\ c \in C;\ \sum_{c\in C} \pi^c = 1. \tag{41}$$

Proof: See below.

The first statement of the lemma is a direct consequence of condition (39). It says that marketer τ takes a position θ_τ^* in financial transmission contracts that results in initial and contingent payments $y_\tau^{0*} = \lambda^{0*}\Gamma^0\theta_\tau^*$ and $y_\tau^{c*} = \lambda^{c*}\Gamma^c\theta_\tau^*$. These payments equalize the first and second stage marginal utilities of the marketer. The second statement of the lemma depends on the existence of a risk free asset and on the no arbitrage assumption; it is an application of the first fundamental theorem of finance theory (e.g. Chapter 1, Duffie (1996)).

Suppose that the θ_τ^* and δ^* exist and satisfy a balance condition $\theta_I^* + \theta_{II}^* + \delta^* = 0$ on the financial market. Using Lemma 1, one can restate the behaviour of marketer τ as obeying the following variational principle.

Seek $e_\tau^* \geq 0$ satisfying

$$\sum_c \left[\alpha_\tau^c L_\tau^c(e_\tau^*, i^{c*}) + \lambda^{0*}\Gamma^0 S\right](e_\tau - e_\tau^*) \geq 0 \text{ for all } e_\tau \geq 0. \tag{42}$$

The interpretation of relation (42) is as follows. Marketer τ pays an up-front cost $\lambda^{0*}\Gamma^0 S e_\tau^*$ for transmission services. This is accounted for with a unitary marginal utility. She also incurs the settlement of energy transactions $L_\tau^c(e_\tau^*, i^{c*})$ at stage 1 in state c. These are accounted with marginal utility α_τ^c. Marketers all pay the same price, $\lambda^{0*}\Gamma^0 S$, for transmission services. These prices are established in the financial market in stage 0. The market is thus physically complete.

This formulation allows one to recover the perfect hedging property put forward in Hogan (2000b) and Ruff (2000). It says that marketers may, if they so wish, perfectly hedge transmission price risk by covering their energy position by nodal forward contracts. As argued in Hogan (2000b) and Ruff (2000) this property allows for changes of PDF because the nodal contracts are entirely defined in terms of the nodal prices $\lambda^0\Gamma^0$ and $\lambda^c\Gamma^c$. Marketers do not have to bother about the network topology and the associated PDFs Γ^0 and Γ^c that prevail in the forward and real time markets.

Relation (42) only provides an intuitive interpretation of the impact of the financial market. A more rigourous formalization of the situation is given by the generalized complementarity Problem XII constructed after adding the additional constraints (40) and (41) to the complementary reformulation of Problem X. In order to correctly interpret Problem XII, recall that α_τ^c now depends on an additional vector θ_τ and that finding this vector is part of the statement of the problem.

Problem XII

Seek $(e^*, (i^{c*}, c \in C))$; λ^{0*}; $(\lambda^{c*} \geq 0, c \in C); y_\tau^{c*} = \lambda^{c*}\Gamma^c\theta_\tau^*, c \in C, \tau = I, II$; $\xi^{c*} \geq 1$, $c \in C$ s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$, $c \in C$, satisfying

$$\left[\sum_c \alpha_\tau^c \left(L_\tau^c(e^*, i^{*c}) + \lambda^{c*}\Gamma^c S\right)\right] \geq 0 \qquad \tau = I, II \tag{43}$$

$$\left[\sum_c \alpha_\tau^c \left(L_\tau^c(e^*, i^{*c}) + \lambda^{c*}\Gamma^c S\right)\right] e_\tau^* = 0 \qquad \tau = I, II \tag{44}$$

$$[O_\tau^c(e^*, i^{*c}) + \lambda^{c*}\Gamma^c] \geq 0, c \in C, \tau = I, II \tag{45}$$

$$[O_\tau^c(e^*, i^{*c}) + \lambda^{c*}\Gamma^c]\, i^{c*} = 0, c \in C, \tau = I, II \tag{46}$$

$$\lambda^{c*}\left[\overline{f}^c - \Gamma^c(Se^* + i^*)\right] = 0, \quad c \in C \tag{47}$$

$$\begin{array}{l} \lambda^{0*}\Gamma^0 = \sum_c \alpha_\tau^c \lambda^{c*}\Gamma^c, \tau = I, II \\ \left(\sum_{c\in C} \xi^{c*}\right) \lambda^{0*}\Gamma^0 = \sum_{c\in C} \xi^{c*}\lambda^{c*}\Gamma^c, \end{array} \tag{48}$$

The constraints (48) added to Problem X force Problem XII to satisfy the conditions (40) and (41) implied by the existence of the financial transmission market. They drastically complicate the analysis of Problem XII. Specifically, it becomes almost impossible to easily derive any statement about existence and uniqueness of a solution. We do not try to prove that Problem XII has a solution but simply assume one. Under this assumption we have the following proposition.

Proposition 10 *Let $(e^*, (i^{c*}, c \in C))$ be part of a solution to Problem XII (assume there exists one). It is also a solution to Problem X after an initial endowment y_τ^{c*}, $c \in C$, generated by the trading of financial contracts has been given to the marketer τ, $\tau = I, II$.*

Proof: See appendix.

One should note that the solutions to Problems X and XII differ by the initial contingent endowments that accrue from the positions taken on the financial market. At the solution to Problem XII, marketer τ indeed pays $\lambda^{0*}\Gamma^0\theta_\tau^*$ in stage zero and receives $\lambda^{c*}\Gamma^c\theta_\tau^*$ in state c of stage 1. These endowments which are absent from Problem X modify the marginal utilities and hence the final equilibrium on the market.

Financial markets are defined as complete or incomplete (Duffie (1996)). A financially complete market is one where the number of traded financial assets allows one to construct a portfolio θ that can reproduce any vector of contingent payoff p^c, $c \in C$. In our context:

Definition 1 *The market of financial transmission contracts is complete iff there always exists a portfolio θ (of initial up front cost $\lambda^{0*}\Gamma^0\theta$) such that*

$$\lambda^{c*}\Gamma^c\theta = p^c, \qquad c \in C \tag{49}$$

for any vector $(p^c, c \in C)$.

Complete markets imply the unicity of risk neutral probabilities. This is the second fundamental result of finance theory (Duffie (1996)). We further elaborate on the completeness of the financial transmission contracts in Section 6.3.3. At this stage we simply state the following lemma.

Lemma 2 *Suppose the market of financial transmission contracts is complete. Then the risk neutral probability is unique and*

$$\eta\pi^c = \alpha_\tau^c, \qquad c \in C, \tau = I, II. \tag{50}$$

where η is a normalization factor.

Proof: See Appendix.

According to Lemma 2, the completeness of financial transmission contracts implies the equality of the marginal utilities of the marketers in each state of the world (relation (50)). This is interpreted by saying that marketers can, together with the speculators, trade the risk that they incur in the different states of the world. This makes it possible for the market to achieve Pareto optimality. In the usual interpretation of finance, π^c is the normalized price of risk in state c. It thus makes sense to define the electricity market as financially complete if the financial transmission contracts are sufficiently diversified to insure that (50) is satisfied. It is indeed only under this condition that the market is Pareto efficient.

The following proposition characterizes the solution of the regional transmission problem when the electricity market is financially complete.

Proposition 11 *Suppose that the market of financial transmission contracts is complete. Then there exists a unique vector of risk neutral probabilities* $\pi^c > 0, \sum_c \pi^c = 1$ *such that the outcome* $e^*, \lambda^{0*}\Gamma_0$ *of the market is the solution to Problem* $XIII$ *defined below.*

Proof: See appendix.

Problem XIII

Seek $(e^*, (i^{c*}, c \in C)), e^* \geq 0$ s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$, $c \in C$, satisfying

$$\begin{aligned}\left[\sum_c \pi^c L_I^c(e^*, i^{c*})\right](e_I - e_I^*) + \left[\sum_c \pi^c L_{II}^c(e^*, i^{c*})\right](e_{II} - e_{II}^*) \\ + \sum_c \left[O_I^c(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^c(e^*, i^{c*})(i_{II}^c - i_{II}^{c*})\right] \geq 0\end{aligned} \tag{51}$$

for all $(e, (i^c, c \in C)), e \geq 0$ s.t. $\Gamma^c(Se + i^c) \leq \overline{f}^c, c \in C$.

6.3.2.2. Discussion

The above discussion is entirely conducted in nodal terms ($\lambda^0\Gamma^0$ and $\lambda^c\Gamma^c$). In order to move to the flow gate interpretation, note that the PDF matrix Γ^0 was only used so far as an intermediate tool for constructing a vector $\lambda^0\Gamma^0$ of nodal prices. Any other choice of λ^0 and Γ^0 that results in the same $\lambda^0\Gamma^0$

would be as good. The situation is different in the flow gate model where contracts are based on line capacities associated to a well defined topology. λ^0 and Γ^0 are separate constituents that both intervene to specify a flow gate transmission contract. In other words the network topology at stage 0 and hence Γ^0 matters. We now examine the financial market associated to the flow gate model. Assume that the relevant contracts are defined in stage 0 on the basis of a given network topology and the associated PDF matrix Γ^0. A financial contract in the flow gate model gives a payoff λ^c for a unitary line capacity in state of the world c when the line is up. The payoff when the line is down is not defined in Chao et al (2000). We therefore complete the definition of these authors by specifying that the contract on a line pays 0 when this line is down.

Because flow gate transmission contracts are different from nodal ones, the marketer's problems need to be revisited. This is done in the following problem.

Problem XIV

Seek $e^*_\tau \geq 0$, θ^* satisfying

$$\sum_c \alpha^c_\tau \left[L^c_\tau(e^*, i^{c*}) + \lambda^{c*}\Gamma^c S\right](e_\tau - e^*_\tau) + \left(\lambda^{0*} - \sum_c \alpha^c_\tau \lambda^{c*}\right)(\theta_\tau - \theta^*_\tau) \geq 0 \quad (52)$$

for all $e_\tau \geq 0, \theta$.

A Problem XIVbis analoguous to Problem XIbis is defined for the speculators.

Lemma 3 is the analogous of Lemma 1 and states

Lemma 3 *A solution to Problem XIV satisfies*

$$\lambda^{0*} = \sum_c \alpha^c_\tau \lambda^{c*}. \quad (53)$$

Moreover, under the no arbitrage assumption, there exists at least one risk neutral probability such that

$$\lambda^{0*} = \sum_{c \in C} \pi^c \lambda^{c*}; \;\; \pi^c > 0, c \in C; \sum_{c \in C} \pi^c = 1. \quad (54)$$

Again, the first statement is a direct consequence of condition (52) while the second statement of the lemma depends on the existence of a risk free asset and the no arbitrage assumption. In contrast with nodal contracts it is not clear how relations (53) and (54) allow one to derive a unique price of transmission services in stage 0. Specifically, relations (53) and (54) do not imply the equality of $\sum_c \alpha_\tau^c \lambda^{c*}\Gamma^c$ for $\tau = I$ and II. This can easily be understood by considering the hedging capabilities of the flow gate transmission contracts. One can indeed easily see that a market that only trades line contracts does not allow marketers to fully hedge their transmission risk when the lines are subject to contingencies. This is stated in the following proposition.

Proposition 12 *Consider a portfolio $\Gamma^0 Se$ of flow gate contracts associated to energy forward transactions e. The uncovered transmission risk in state c is equal to $\lambda^c(\Gamma^0 - \Gamma^c)Se$. This risk vanishes when there are no line contingencies.*

Proof: See appendix.

It appears that transmission risks cannot be fully hedged by the sole financial flow gate contracts, except when there are no contingencies. This implies that, except for this case, we are unable to equalize the $\sum_c \alpha_\tau^c \lambda^{c*}\Gamma^c$ of the different marketers. In consequence we are unable to find a unique transmission price in stage 0. This makes the market physically incomplete.

A relevant question is whether this market incompleteness is sufficiently important to make the flow gate model unpractical, and, if so, whether other financial contracts will not be proposed to complete the market. Alternatively because the residual risk is really a "volume risk" due to reconfiguration of the network, one could envisage that this risk is covered by the ISOs (Chao and Peck (1998), Wilson (1997)). In any case it would seem from this analysis that the flow gate system suffers from hedging defects that the nodal system does not have. As we discuss in the following section the situation is not as clear cut.

6.4 More on market completeness

The above discussion may clarify some of the statements of the proponents of the nodal model. It is indeed ascertained that one of the major advantages of the nodal system is that it allows for perfect hedging of transmission prices. This certainly requires some condition: perfect hedging is only possible when a sufficiently rich set of financial contracts exist. Specifically a marketer who wants to hedge a transaction between two nodes cannot realistically do so if the market of financial contracts between these two nodes does not exist or is not sufficiently liquid. Absence of one of these conditions results in missing or thin markets and hence leads to a physically incomplete transmission market.

As seen before perfect hedging requires an additional condition in the flow gate model, namely that network contingencies are excluded. This is too harsh to be realistic. Perfect hedging will thus never be possible. This phenomenon has been pointed out by Hogan (2000b) and Ruff (2000). It is certainly a disadvantage with respect to the nodal system but possibly not a too serious one. As noted before, physical completeness in the nodal system is indeed only guaranteed if there are enough traded financial transmission contracts. This may also be a difficult condition to fulfill in practice. In any case incomplete markets are commonly accommodated in the real world provided they are not too damaging. It may be that the hedges provided by flow gate contracts are sufficient. Similarly it may be that a subset of the theoretically strictly necessary nodal contracts suffices. In fact we do not really know.

Even though the nodal model may exhibit advantages with respect to flow gates when it comes to physically completing the market, it may be a surprise to note that none of the above models and organizations will meet the financial completeness criterion. Indeed the auctionning of transmission services insures the equality of transmission prices but not of the marginal utilities of traders in different states of the world. Similarly it is unlikely that the nodal and flow gate transmission financial contracts will be numerous enough to achieve this equality. We may thus end up with the conclusion that none of the proposed systems is powerful enough to financially complete the market.

This implies that risk will always be handled in a non Pareto optimal way, whether perfect hedging of transmission transactions is possible or not. This raises the question of whether there is any hope to achieve financial completeness and hence Pareto optimality. A further analysis of the risks involved may shed some light on the issue.

Part of the attractiveness of the nodal model is that it aggregates all the electricity risks of a bilateral transaction into one. In other words, market risks such as demand and supply uncertainties as well as network risks such as line defaults are aggretated into nodal price risk. But these risk factors are distinct in the real world. Moreover, they are not of the same type. Typically a line risk is like a jump while daily demand uncertainties look more like a diffusion. The flow gate model unbundles some of these risks but fails to provide enough instruments to cover all of them. Interestingly the debate has centered on which system would lead to the smallest number of tradable instruments. The question is unanswered at this stage. But there is an other question. Given the different packaging of risk hedges offered by the nodal and flow gate models and the unequal coverage that they allow, will the market prefer a system that offers full coverage of bundles of risks to one that provides individual coverage of only some riks factors ?

This suggests an alternative perspective for looking at the nodal and flow gate models. The objective of financial engineering is not necessarily to provide the smallest possible set of instruments that cover global risks. Its aim is to untangle risks factors and to re-bundles them in packages that are attractive to agents operating in the market. In this sense derivative products that offer perfect hedging may not necessarily be what the market wants most. Electricity is a very volatile commodity with a lot of underlying risk factors. Maybe a first question is to see the extent to which one can define elementary derivative products that span each or at least sufficiently small subsets of these risk factors. The next question would then be to re-bundle them in products that the market may desire. The discussion about the optional nature of flow gate versus nodal contracts, which has been systematically avoided in this paper is of immediate relevance in this context. It is a well-known property that options can be used to complete markets that are initially (financially) incomplete. This is done by introducing options with different strike prices. The flow gate model has so far only introduced options with zero strike price. It therefore did not take full advantage of the optional nature of the contracts that it offers. An analysis of the capability of the flow gate contracts to better span the domain of risk factors of the electricity market is beyond the scope of this paper, but the following last property may shed some line on what is at stake. We did indicate before that the flow gate model in its current form offered inferior hedging properties compared to the nodal system. Suppose that one is able to enlarge the set of instruments traded in the market (for instance by offering flow gate contracts with non-zero strike prices or equivalently with different priorities). Then the following result could be of relevance.

Proposition 13 *Suppose the market of financial flow gate contracts is complete. Then the marginal utilities of the traders are equal in each state of the world and the flow gate contracts allow full hedging of transmission risks. Moreover the price for transmission paid by the marketers is equal in stage 0 and the market is physically complete.*

Proof: See appendix.

The relevant question thus appears to identify which of the nodal or flow gate, or of the combination of both models is the most capable of producing the bundle of derivative products that the market wants most to trade the numerous risk factors inherent to electricity.

7 Conclusions

The restructuring of electricity is certainly a difficult process. This should not be a surprise. Electricity was painful to regulate; there is no reason to believe that it can easily be deregulated. In short electricity is a difficult product. We here explore some of the questions arising in the design of regional transmission systems consisting of different control areas operating in interaction. This problem is relevant both to the US and Europe. Our goal is to concentrate on different degrees of market incompleteness that may be burried in proposals of regional transmission system. Because the nodal and flow gate proposals are the most structured proposals in presence, we conduct the discussion by reference to these two models, with as background current proposals for the organization of cross border trade in Europe. After recalling some basic notions in Section 2, we compare in Section 3 an organization without market of transmission services to one with transmission markets in nodal or flow gate form. Even though the need for transmission markets is well accepted, criticisms against the more formalized models persist. Specifically excessive complexity is often invoked against the nodal system. The flow gate model is an attempt to propose a "simpler" organization of the market for transmission services. Even this seems too complicated for the European Association of Transmission Operators which keeps proposing to work with aggregate transmission capacities. We therefore try to come up with a sound definition of this notion. Needless to say a definition cannot waive the inherent complexity of the network. But it can bundle some of this complexity into a set of tasks that can be assigned to the ISOs. Section 4 therefore introduces the notion of an aggregate or extended flow gate. This leads to a dual vue of the network: marketers procure aggregate transmission capacities to conduct their energy transactions and ISOs assemble these transmission capacities from line and ancillary services. This maintains a physically complete market. The need to conduct a security constrained dispatch has been argued against the flow gate proposal. We show in Section 5 that our dual view of the market is robust with respect to this criticism. Section 6 extends the previous discussion to the situation where there exist both a forward and spot market. Energy and transmission transactions are concluded in the forward market and settled in the spot market. We analyze this organization in two ways. In the first approach, transmission capacities are initially auctionned in the forward market. We show that physical market completeness is achieved both in the nodal and flow gate models, provided the latter allows for aggregate flow gates. The second approach introduces financial transmission contracts. We show that these physically complete the market in the nodal system but that the same results can only be obtained under additional assumptions in the flow gate system. Even though trading transmission capacities (whether through physical or financial contracts) makes the market physically complete (possibly under certain conditions), it does not allow the marketers to fully trade the risks that they are subject to. Specifically the marginal utility of money of marketers can still differ in the states of the world. The market is thus financially incomplete.

It may come to a surprise that financial completeness also seems difficult to achieve with financial transmission contracts. The problem is that electricity involves too many risk factors. Spanning the relevant uncertainty implies a large set of derivative products that may not be immediately at hand. The challenge therefore seems to be in identifying the right set of derivatives products that the market wants in order to trade risk. The flow gate model, because of the natural optional character of the contracts that it defines seems better fit in this respect but this certainly deserves more exploration. In any case making the market of financial transmission contracts complete, whether we are talking in nodal or flow gate restore the two models on a equal foot.

Appendix

Proof of Proposition 1. Relation (6) follows from Theorem 4 in Harker (1991). The existence of a solution and the likely multiplicity of the solutions are general results of quasi-variational inequalities found in the same paper.

Proof of Propositions 3, 4, 5 and 6. The proof of these propositions results from the application of the extension of KKT conditions to variational inequalities (Harker and Pang (1990)). Existence follows from Assumption 2.

Proof of Proposition 8. Define $O^c(e^*, i^c) = (O^c_I(e^*, i^c_I), O^c_{II}(e^*, i^c_{II}))$. By construction each component of $O^c(e^*, i^c)$ is either a nodal supply or demand curve. $O^c(e^*, i^c)$ is thus integrable in the sense of Harker and Pang (1990) and its integral is the sum of the generation costs at generations nodes minus the willingness to pay at consumption node. The solution to problem $VIII^{2,c}$ is thus also the solution to the optimization problem that maximizes this function subject to the network constraint $\Gamma^c(Se^* + i^c) \leq \overline{f}^c$.

Proof of Proposition 9. Problem IX^{2c} is equivalent to the pair of problems. Seek i_τ^{c*} s.t. $\Gamma^c(Se^* + i_\tau^{c*} + i_{\tau' \neq \tau}^{c*}) \leq \overline{f}^c$ satisfying

$$O_\tau^c(e^*, i_\tau^{c*})(i_\tau^c - i_\tau^{c*}) \geq 0$$

for all i_τ^c s.t. $\Gamma^x(Se^* + i_\tau^c + i_{\tau' \neq \tau}^{c*}) \leq \overline{f}^c$. Using the same reasoning as in the proof of Proposition 8, each of these problems is equivalent to a single dispatch optimization problem where each ISO tries to relieve congestion at minimal cost. This obviously is not the same as both ISOs jointly minimizing costs to relieve congestion. The multiplicity of solutions derives from the same application of the results of Harker (1991) as the proof of Proposition 1.

Proof of Proposition 10. Note that (44), (45), (46), (47) constitue the complementarity reformulation of (35) after adding the multipliers λ^{c*} (Harker and Pang (1990)). (48) is the reformulation of (40) and (41) where $\xi^c \geq 1$ is introduced to insure that $\pi^c = \dfrac{\xi^c}{\sum_{c'} \xi^{c'}} > 0$.

Proof of Lemma 2. The uniqueness of the risk neutral probability is the second fundamental result of finance (e.g. Duffie, 1996). The equality of α_τ^c to Γ^c (up to some constant factor) derives from the equilibrium condition obtained from marketers trading risks in a complete financial market.

Proof of Proposition 11. Completeness of the financial transmission contracts implies that (48) in Problem XII is automatically insured by the financial market. Problem XII is then rewritten as

Seek $(e^*, (i^{c*}, c \in C))$, λ^{0*}, $(\lambda^{c*} \geq 0, c \in C)$, s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$, $c \in C$ satisfying

$$\left[\sum_c \pi^c \left(L_\tau^c(e^*, i^{c*}) + \lambda^{c*}\Gamma^c S\right)\right] \geq 0, \tau = I, II$$

$$\left[\sum_c \pi^c \left(L^c_\tau(e^*, i^{*c}) + \lambda^{c*}\Gamma^c S\right)\right] e^*_\tau = 0 \qquad \tau = I, II$$
$$[O^c_\tau(e^*, i^{*c}) + \lambda^{c*}\Gamma^c] \geq 0, c \in C, \tau = I, II$$
$$[O^c_\tau(e^*, i^{*c}) + \lambda^{c*}\Gamma^c]\, i^{c*} = 0, c \in C, \tau = I, II$$
$$\lambda^{c*}\left[\overline{f}^c - \Gamma^c(Se^* + i^{*c})\right] = 0, \quad c \in C.$$

These expressions are the complementarity form of Problem $XIII$ after adding the multipliers λ^{*c} (Harker and Pang (1990)) and multiplying the constraints by π^c.

Proof of Proposition 12. Let $\theta = \Gamma^0\, Se$ be the portfolio of financial flow gate contracts subscribed to cover transaction e. In contingency c, this contract pays $\lambda^c\Gamma^0 Se$ while the transmission cost is $\lambda^c\Gamma^c Se$. Hence the result.

Proof of Proposition 13. Using the same reasoning as for nodal contracts completeness of the financial transmission contracts implies a single risk neutral probability such that $\xi\pi^c = \alpha^c_\tau$, $\tau = I, II, c \in C$. This implies that $\sum_c \alpha^c_\tau \lambda^{c*}\Gamma^c$ is equal for both marketers and hence that they can fully hedge the transmission risk in stage 0.

References

[1] Birge, J.R. and F. Louveaux (1997). *Introduction to Stochastic Programming.* New York, Springer.

[2] Boucher, J. and Y. Smeers (2000). Alternative models of restructured electricity systems. Part 1: No market power. Forthcoming in *Operations Research.*

[3] Boucher, J. and Y. Smeers (2001). Comments on ETSOs proposal for congestion management. Mimeo, CORE, Université catholique de Louvain, Louvain-la-Neuve, Belgium.

[4] Boucher, J., Ghilain, B. and Y. Smeers (1998). Security-constrained dispatch gives financially and economically significant nodal prices. *The Electricity Journal*, November 1988, 53–59.

[5] Cadwalader, M.D., Harvey, S.M., Hogan, W.W. and S.L. Pope (1999). Coordinating congestion relief across multiple regions. Center for Business and Government, J.F.K. School of Government, Harvard University, Cambridge (Mass.), October 1999.

[6] Chao, H.-p. and S. Peck (1996). A market mechanism for electric power transmission. *Journal of Regulatory Economics*, 10, 25–60.

[7] Chao, H.-p. and S. Peck (1998). Reliability management in competitive electricity markets. *Journal of Regulatory Economics*, 3, 189–200.

[8] Chao, H.-p., Peck, S., Oren, S. and R. Wilson (2000). Flow based transmission rights and congestion management. Forthcoming in *The Electricity Journal*.

[9] Daxhelet, O. and Y. Smeers (2000). Variational inequality models of restructured electricity systems. Forthcoming in M.C. Ferris, O. L. Mangasarian and J.-S. Pang (eds.), *Applications and Algorithms of Complementarity*, Kluwer Academic Publishers.

[10] Daxhelet, 0. and Y. Smeers (2001). Introducing reliability markets in models of restructured electricity systems. Mimeo, CORE, Université catholique de Louvain, Louvain-la-Neuve, Belgium.

[11] Debreu, G. (1952). A social equilibrium existence theorem. *Proceedings of the National Academy of Sciences U.S.A.*, 38, 886–893.

[12] Duffie, D. (1996). *Dynamic Asset Pricing Theory.* Princeton (New Jersey), Princeton University Press.

[13] ETSO (1999). Evaluation of congestion management methods for cross-border transmission. European Association of Transmission System Operators.

[14] Hanger, J. (2000). The perfect RTO ? Even at PJM; pulling it off is no cinch. *Public Utilities Fortnightly*, May 1, 2000, vol. 138(9), 34–37.

[15] Harker Patrick T (1991). Generalized Nash Games and Quasi-Variational Inequalities. *European Journal of Operations Research*, 54, 81-94.

[16] Harker P. T. and J.S. Pang (1990). Finite-dimensional variational inequality and non-linear complementarity problems: a survey of theory, algorithm and applications. *Mathematical Programming*, 48B, 161-220.

[17] Harvey, S.M., Hogan, W.W. and S. Pope (1997). Transmission capacity reservations and transmission congestion contracts. Center for Business and Government, Harvard University, Cambridge (Mass.).

[18] Hobbs, B.F. (2001). Linear complementarity models of Nash Cournot competition in bilateral and POOLCO power markets. Forthcoming in *IEEE Transactions on Power Systems.*

[19] Hogan, W. (1992). Contract networks for electric power transmission. *Journal of Regulatory Economics*, 4, 211-242.

[20] Hogan, W. (2000a). Regional transmission organizations: Millenium order on designing market institutions for electric network systems. Center for Business and Government, John F. Kennedy School of Government, Harvard University, Cambridge (Mass.), May 2000.

[21] Hogan, W. (2000b). Flow gates rights and wrongs. Center for Business and Government, J.F.K. School of Government, Harvard University, Cambridge (Mass.), August 2000.

[22] Kempfert, C. (2000). Modelling an oligopolistic German electricity market structure. Investigation of a Nash equilibrium by a computational game theoretic modelling tool. Paper presented at the "Annual European Energy Conference 2000, IAEE/SNF, Bergen, Norway, August 31–September 1, 2000.

[23] Palmeri, C. (2001). California; it didn't have to be this way. *Business Week*, p. 44, January 22, 2001.

[24] Rivier, M., Ventara, M. Ramos, A. Martinez-Corcoles, F. and A. Chiarri Toscano (2000). A generation operation planning model: Deregulated electricity markets based on the complementarity problem. Forthcoming in M.C. Ferris, O. L. Mangasarian and J.-S. Pang (eds.), *Applications and Algorithms of Complementarity*, Kluwer Academic Publishers.

[25] Rosen, J.B. (1965). Existence and uniqueness of equilibrium points for concave n-person game. *Econometrica*, 33, 520–534.

[26] Ruff, L.R. (2000). Flow-gates, Contingency, Constrained dispatch and FTRs. Available from http://www.stanford.edu/group/EMF/meet.

[27] Schweppe, F.C., Caramanis, M.C. Tabors, R.E. and R.E. Bohn (1988). *Spot pricing of electricity*. Boston, Kluwer Academic Publishers.

[28] Stavros, R. (2000). Transmission 2000: Can ISOs iron out the seams ? *Public Utilities Fortnightly*, May 1, 2000, vol. 138(1), 24–33.

[29] Wilson, R. (1997). Implementation of priority insurance in power exchange markets. *Energy Journal*, 18(1), 111-123.

[30] Wilson, R.B. (1999). Market architecture. Mimeo, Stanford University, Stanford.