

Strategic Behavior and the Pricing of Gas in Mexico*

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Abstract

This paper looks at various models that address strategic behavior in the supply of gas. The paper has three very strong technical results. First, the netback pricing rule leads to discontinuities in Pemex's revenue function. Second, having Pemex pay for the gas it uses and the gas it flares increases the value of the Lagrange multiplier associated with the gas processing constraint. Third, if the gas processing constraint is binding, having Pemex pay for the gas it uses and flares does not change the short run optimal solution for the optimization problem so it will have no impact on short-run behavior.

JEL classification: L51

Key words: natural gas, strategic pricing, benchmark regulation, Mexico

I. Introduction to Problem

The initial economy studies of the efficiency of the netback rule were done under the assumption that the gas at Ciudad Pemex was produced as a joint product with oil and that Pemex did not behave strategically in the short run in supplying that gas to market. It was noted that in the long run there existed incentives for Pemex to shift the arbitrage point south, but at the time this did not seem like an important issue as a substantial amount of gas from Ciudad Pemex was reaching Los Ramones and there was little

* The research reported in this paper was supported by the Comisión Reguladora de Energía through a grant to Centro de Investigación y Docencia Económicas (CIDE). The second author also acknowledges support from the Repsol-YPF-Harvard Kennedy School Fellows program, and the Fundación México en Harvard.

incentive for short run strategic behavior. Since that time, things have changed. Demand for gas in the south of Mexico has increased and the capacity of the pipelines connecting Mexico with the United States pipeline system has also increased. The arbitrage point may shift south to Cempoala. Thus, at this point there may some incentives for Pemex to behave strategically in supplying gas to the Mexican market. This paper is a study of the implications of such behavior and possible instruments that can be used to eliminate possible inefficiencies. The paper will look at different models of the varying complexity to study this problem.

The Mexican pipeline system can be modeled as a line on the interval $[0,b]$ with a distribution function $f(n)$ that has mass points at the border with Texas, Los Ramones, and Cempoala.¹ The pipelines also have intervals where the demand is zero. For convenience we will refer to any open interval (n_i, n_j) where $f(n) = 0$ as a *gap* in the distribution.

The point of arbitrage is defined as the point where gas from Ciudad Pemex in the south meets the gas from Burgos and Texas. Assume that the distance between the Texas border and Ciudad Pemex is \bar{d} , and the total demand is given by

$$(1) \quad \bar{Q} = \int_0^{\bar{d}} f(n)dn.^2$$

¹ See Brito and Rosellon (2002)

² We will assume that the demand of individuals for gas is not a function of price. This assumption is made for simplicity and does not change any of the results. We are also assuming that the pipeline system is not a binding constraint in Pemex supplying gas to market. This is a valid assumption at the moment, but it should be noted that the feasibility of the netback rule to serve as a pricing mechanism for gas in Mexico depend on gas being able to move freely to equilibrate markets.

Assume that Pemex supplies Q amount of gas to the market. Then distance from the arbitrage point to Ciudad Pemex is given by the solution of

$$(2) \quad Q = \int_0^{\bar{d}} f(n) dn$$

which we will define as $d(Q)$. The price of gas at a point n at the present time is given by:

For all gas north of the arbitrage point, $n > d(Q)$, the price of gas is the price at Houston plus the transport cost

$$(3) \quad p = p_h + c(\bar{d} - n)$$

For all gas south of the arbitrage point, $n < d(Q)$, the price of gas is the price at the arbitrage point less the transport cost.

$$(4) \quad p = p_h + c(\bar{d} - d) - cn$$

Thus the price of gas at Ciudad Pemex is given by

$$(5) \quad p_q = p_h + c\bar{d} - 2cd(Q)$$

so the price of gas at Ciudad Pemex is a function of Q , $p_q(Q)$. Note that we are assuming that individual demands are not responsive to prices. The downward sloping demand curve faced by Pemex is strictly a function of the net back rule.

In general, the demand curve faced by Pemex under the pricing rule is downward sloping in regions where the demand is positive and there are no mass points. This is because increasing sales move the point of arbitrage north. The price is constant in intervals of demand that correspond to mass points. This is because Pemex can sell more gas without

moving the point of arbitrage. Finally, there are intervals in the pipeline where there are gaps. The demand curve faced by Pemex is discontinuous at these points; an infinitesimal shift in supply will move the point of arbitrage by a substantial amount and this leads to a discontinuity.

II. Fixed supply of gas model.

We will initially assume that the amount of pipeline quality gas Pemex produces is \bar{X}_1 . This amount will be assumed to be fixed and not under the control of Pemex. Let Q be the amount of gas Pemex actually supplies to the market. We will investigate the optimal sales policy for Pemex under the assumption that it is maximizing profits for a distribution function that is uniform and a distribution function that has mass points. These assumptions are made to simplify the exposition and do not change any of the substantial results. The more general case is addressed later in the paper.

Uniform Distribution

Assume that the distribution of demand is uniform and $f(n) = \gamma$. Then

$$(6) \quad d = \frac{Q}{\gamma}$$

and

$$(7) \quad p_q = p_h + c\bar{d} - \frac{2cQ}{\gamma}$$

for all $Q < \bar{Q}$ and $p_q = p_h - c\bar{d}$ for all $Q \geq \bar{Q}$. For simplicity we are ignoring the cost of transport between Houston and the border. The demand and marginal revenue curve are given in Figure 1 below.

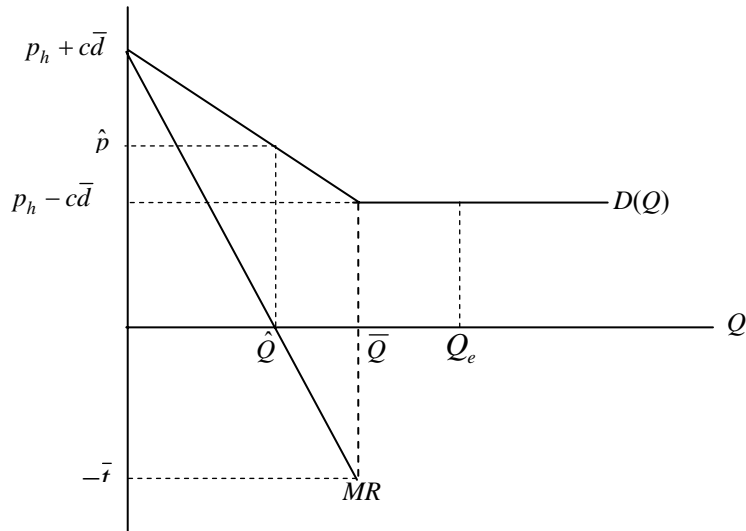


Figure 1

In Figure 1 the quantity Q_e is the point where the amount of gas is sufficient for Pemex to maximize revenue by exporting to the United States. If $Q < Q_e$, then Pemex can maximize revenue by supplying \hat{Q} at a price \hat{p} . If $Q \geq Q_e$, then Pemex can maximize revenue by exporting gas at a price $p_h - c\bar{d}$. Note that the marginal revenue is discontinuous at \bar{Q} where it goes from $-\bar{t} < 0$ to $p_h - c\bar{d} > 0$

Assume that Pemex has an amount of gas \bar{X}_1 it can supply to the market and define Q_f as flared gas, Pemex would maximize

$$(8) \quad \pi(Q, Q_f) = (p_h + c\bar{d} - \frac{2cQ}{\gamma})Q$$

subject to

$$(9) \quad Q + Q_f = \bar{X}_1$$

The Lagrangian is

$$(10) \quad L = (p_h + cd - \frac{2cQ}{\gamma})Q + \lambda(\bar{X}_1 - Q - Q_f)$$

and the first order conditions with respect to Q and Q_f are:

$$(11) \quad (p_h + cd - \frac{4cQ}{\gamma}) - \lambda = 0$$

$$(12) \quad \lambda > 0 \quad \lambda Q_f = 0$$

There are three cases. First if

$$(13) \quad (p_h + cd - \frac{4c\bar{X}_1}{\gamma}) \geq 0$$

then $\lambda > 0$ and all the gas will be supplied to the market. Second, if

$$(14) \quad (p_h + cd - \frac{4c\bar{X}_1}{\gamma}) < 0,$$

and $Q < Q_e$. Then $\lambda = 0$ and Pemex will flare gas and supply an amount of gas that satisfies the condition

$$(15) \quad (p_h + cd - \frac{4cQ}{\gamma}) = 0.$$

It is clear in this case that if Pemex is maximizing profits and behaves strategically, then gas will be withheld from the market and flared.

The third case is if $\bar{X}_1 \geq Q_e$. Then $(p_h - c\bar{d})\bar{X}_1 \geq \hat{p}\hat{Q}$ and Pemex would export all the gas not consumed in Mexico.

One instrument that would reduce the incentive to flare gas in the second case is to impose a tax on flared gas. Suppose a tax on flared gas was imposed on Pemex, then Pemex it would maximize

$$(16) \quad \pi(Q, Q_f) = (p_h + cd - \frac{2cQ}{\gamma})Q - tQ_f$$

subject to

$$(17) \quad Q + Q_f = \bar{X}_1$$

The Lagrangian is

$$(18) \quad L = (p_h + cd - \frac{2cQ}{\gamma})Q - tQ_f + \lambda(\bar{X}_1 - Q - Q_f),$$

and the first order conditions with respect to Q and Q_f are:

$$(19) \quad (p_h + cd - \frac{4cQ}{\gamma}) - \lambda = 0$$

$$(20) \quad -\lambda - t \leq 0 \quad -(\lambda + t)Q_f = 0$$

There are two possible solutions. First if

$$(21) \quad (p_h + cd - \frac{4c\bar{X}_1}{\gamma}) \geq 0,$$

then all the gas available will be supplied to the market. Second, if

$$(22) \quad (p_h + cd - \frac{4c\bar{X}_1}{\gamma}) - t \geq 0,$$

gas will be flared.

However, since t is a policy variable, it can be chosen such that $t > \bar{t}$ and Pemex would not withhold gas from the market. (See Figure 1) The discontinuity at \bar{Q} does not create any problems because marginal revenue increases from the level $-\hat{t}$. Thus, if the domestic distribution of demand is continuous, then it is possible to regulate the supply of gas by imposing a linear tax on flaring gas. This result depends on the distribution of demand not having mass points and gaps.

Mass points and Gaps

Now let us assume that the distribution has mass points at Cempoala, Los Ramones and Houston. The distribution is zero elsewhere. Assume the demand at Cempoala is Q_c and the demand at Los Ramones is Q_r . Further, assume Pemex is a price taker in the Houston market and can sell any quantity of gas at a price p_h . The demand curve for gas at Ciudad Pemex is given by

$$(23) \quad p_c = p_h + c(d_h - 2cd_c) \quad \text{for all } Q \leq Q_c$$

$$(24) \quad p_r = p_h + c(d_h - 2cd_r) \quad \text{for all } Q \leq Q_c + Q_r$$

$$(25) \quad p_b = p_h - cd_h \quad \text{for all } Q > Q_c + Q_r$$

The demand curve is illustrated in Figure 2.

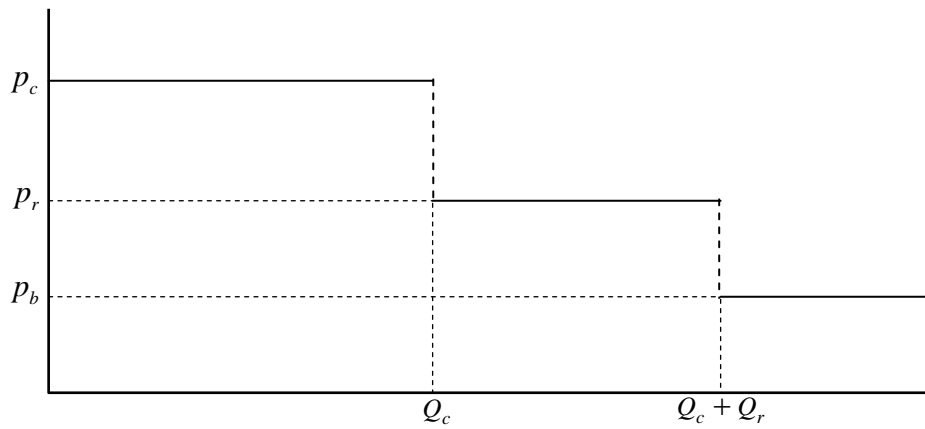


Figure 2

Note that the derivative of the demand function is undefined at Q_c and $Q_c + Q_r$

The revenue function $\pi(Q)$ associated with this demand curve is

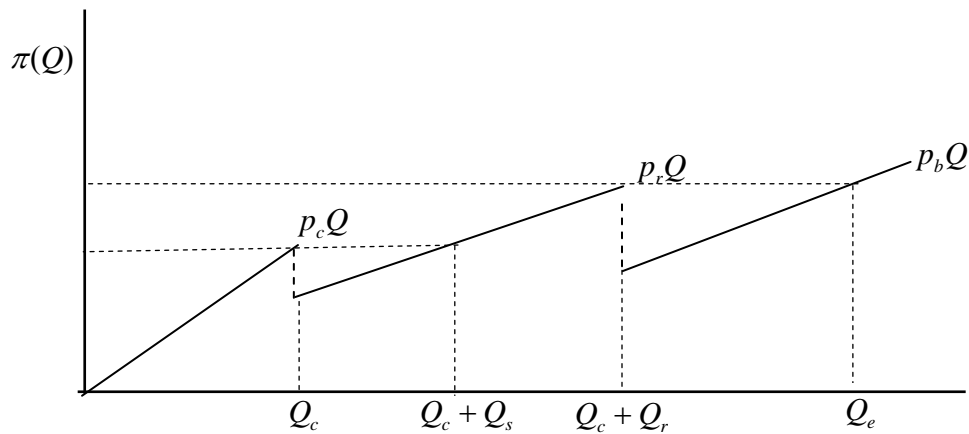


Figure 3

As seen in Figure 3, there are discontinuities in the revenue function at $Q = Q_c$ and at $Q = Q_c + Q_r$. Define $Q_k = Q_c + Q_s$. The value of Q_s is defined such that $p_r Q_k = p_c Q_c$ and

Pemex is indifferent between supplying and amount Q_c at a price p_c and an amount Q_k at a price p_r . Thus, Pemex will not flare gas if the amount available is greater than Q_k .

If the amount of gas Pemex has available is in the interval $[0, Q_c)$ or in the interval $[Q_c + Q_s, \infty)$, the maximization problem is simply

$$(26) \quad \pi(Q, Q_f) = p_i Q - t Q_f \quad i=c, a$$

subject to

$$(27) \quad Q + Q_f = \bar{X}_1$$

Pemex will clearly sell all the gas it has, as it is a price taker and the objective function is locally concave. However, in the interval $[Q_c, Q_c + Q_s)$ the discontinuity in the objective function creates a problem and the problem cannot be solved using the standard optimization techniques such as the Kuhn-Tucker Theorem. Pemex has to supply more than $Q_c + Q_s$ before revenues are greater than $p_c Q_c$. Marginal revenue at the points of discontinuity is $-\infty$ and a tax on the flaring of gas will not work. If Pemex has $Q < Q_c + Q_s$ available, then it will supply the gas to market only if

$$(28) \quad p_c Q_c - t Q_f = p_r (Q_c + Q_f)$$

or

$$(29) \quad t = \frac{p_r (Q_c + Q_f) - p_c Q_c}{Q_f}$$

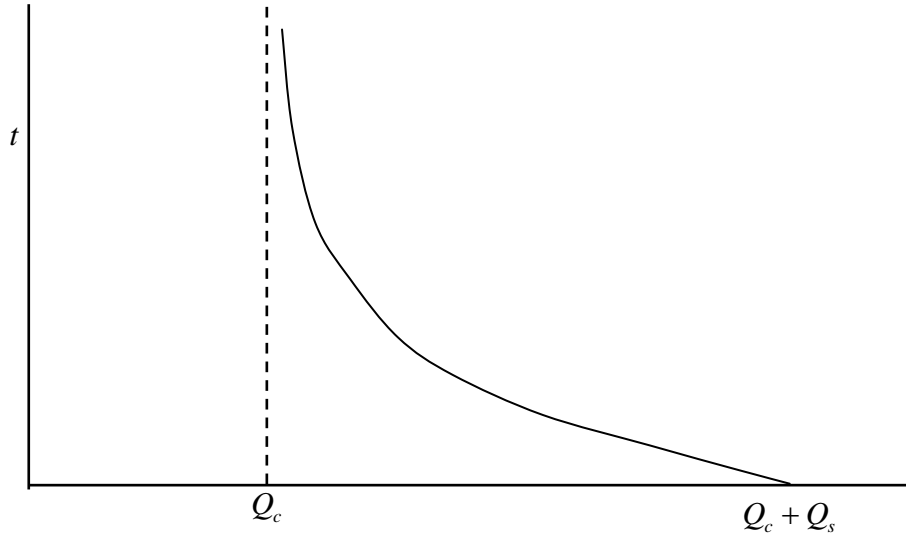


Figure 4

As illustrated in Figure 4, the tax on flaring gas would have to be very large if the amount of gas available is not much larger than Q_c . The tax would not have any relationship to the opportunity cost of the gas. It would likely be politically very difficult to implement. However, a policy that would induce Pemex not to withhold gas from the market can be implemented by defining the arbitration point by the amount of gas Pemex has the *potential* to deliver. Thus the price is defined by \bar{X}_1 which we have assumed is not under the control of Pemex. We will now drop this assumption and assume that Pemex has some control over the amount of gas available.

III. Joint Production of Gas and Oil

We will now drop the assumption that the amount of gas Pemex has available to supply the market is fixed and assume that pipeline quality gas is a joint product with the

production of oil, Z . We will also assume that the price of gas at Ciudad Pemex is given by a general demand function of the form

$$(30) \quad p_q = P(Q)$$

The short run production function for oil and gas is given by

$$(31) \quad Z = F(X_2)$$

$$(32) \quad X_1 = \beta F(X_2)$$

where Z is the oil produced, X_1 is the total amount of gas produced, and X_2 is the gas that is used to produce oil and gas. β is a constant that give the proportion between oil and gas. Let p_o be the price of oil and c be the cost of energy. Then Pemex would want to maximize the revenue from the sale of oil and gas less the cost of production.

$$(33) \quad \pi = P(Q)Q + p_o Z$$

subject to the production constraints

$$(34) \quad \beta F(X_2) - Q - Q_f - X_2 = 0$$

$$(35) \quad F(X_2) - Z = 0$$

The Lagrangian is

$$(36) \quad L = P(Q)Q + p_o Z + \lambda_1[\beta F(X_2) - Q - Q_f - X_2] + \lambda_2[F(X_2) - Z].$$

Where λ_1 is the Lagrange multiplier associated with the production of gas and λ_2 is the Lagrange multiplier associated with the production of oil. The first order conditions are:

$$(37) \quad P(Q) + \frac{dP(Q)}{dQ}Q - \lambda_1 = 0$$

$$(38) \quad \lambda_1 \geq 0 \quad \lambda_1 Q_f = 0$$

$$(39) \quad p_o - \lambda_2 \leq 0 \quad Z[p_o - \lambda_2] = 0$$

$$(40) \quad (\lambda_1 \beta + \lambda_2) \frac{dF(X_2)}{dX_2} - \lambda_1 = 0$$

If $\frac{dp(Q)}{dQ} = 0$, then Pemex will behave as a price taker and there are no problems.

Problems can occur if $\frac{dP(Q)}{dQ} > 0$ or if $\frac{dP(Q)}{dQ}$ is undefined because of a discontinuity.

Let us first consider the case where $\frac{dP(Q)}{dQ} > 0$. There are two possible solutions. First if

gas is not flared and $Q_f = 0$, then $\lambda_1 > 0$ and all the gas will be supplied to the market.

Second, if gas is flared, then $Q_f > 0$, and $\lambda_1 = 0$.

Let us consider the case where $\lambda_1 > \tilde{0}$. Then

$$(41) \quad \lambda_1 = P(Q) + \frac{dP(Q)}{dQ} Q < p_q$$

and

$$(42) \quad \frac{dF(X_2)}{dX_2} = \frac{\lambda_1}{(\lambda_1 \beta + p_o)} < \frac{p_q}{(p_q \beta + p_o)}$$

if $\lambda_1 < p_q$. This implies that Pemex will use more than the optimal amount of gas in the production of gas and oil. This is because the shadow price of gas to Pemex is the marginal revenue rather than the market price.

Now suppose that gas is flared, then $\lambda_1 = 0$ then Pemex treats gas as a free good and

$$(43) \quad \frac{dF(X_2)}{dX_2} = 0 < \frac{P_q}{(p_q\beta + p_o)}$$

Strategic behavior will result in Pemex using too much gas in the production of oil. What is happening is that the shadow price oil is set equal to marginal revenue rather than price. Denote the solution by \hat{Q} .

A possible way to reduce the amount of gas Pemex consumes is to have Pemex pay the market price for the gas it uses. In that case Pemex would want to maximize the revenue from the sale of oil and gas less the cost of production where the cost of production includes the cost of gas used in the production of gas and oil as well as flared gas.

$$(44) \quad \pi = P(Q)(Q - Q_f - X_2) + p_o Z$$

The Lagrangian is

$$(45) \quad L = P(Q)(Q - Q_f - X_2) + p_o Z + \lambda_1[\beta F(X_2) - Q - Q_f - X_2] + \lambda_2[F(X_2) - Z]$$

The first order conditions are:

$$(46) \quad P(Q) + \frac{dP(Q)}{dQ}(Q - Q_f - X_2) - \lambda_1 = 0$$

$$(47) \quad -P(Q) - \lambda_1 \leq 0 \quad [P(Q) + \lambda_1]Q_f = 0$$

$$(48) \quad p_o - \lambda_2 \leq 0 \quad Z[p_o - \lambda_2] = 0$$

$$(49) \quad (\lambda_1\beta + \lambda_2)\frac{dF(X_2)}{dX_2} - P(Q) - \lambda_1 = 0$$

or

$$(50) \quad [\lambda_1(\beta-1) + p_o] \frac{dF(X_2)}{dX_2} - P(Q) = 0$$

Let us first consider the case where gas is not flared. In that case $Q_f = 0$ implies that

$$(51) \quad P(Q) + \frac{dP(Q)}{dQ}(Q - X_2) - \lambda_1 = 0$$

Denote the solution by \tilde{Q} . The amount of distortion depends on the magnitude of the

term, $\frac{dP(\tilde{Q})}{dQ}(\tilde{Q} - \tilde{X}_2)$. If, as is the current case, $0 > \frac{dP(\tilde{Q})}{dQ}(\tilde{Q} - \tilde{X}_2) > \frac{dP(\hat{Q})}{dQ}\hat{Q}$, the

distortion will be smaller, but the shadow price of gas will be different from the market price. If $(\tilde{Q} - \tilde{X}_1) < 0$, it is even theoretically possible that Pemex will supply too much gas to the market. If Pemex were consuming more gas than it supplied to the market, it would be in its self-interest to lower the price of gas.

Now suppose that gas is flared. Then from the Kuhn-Tucker condition given by (47)

$\lambda_1 = -P(\tilde{Q})$. Then equation (51) can be written as

$$(52) \quad 2P(\tilde{Q}) + \frac{dP(\tilde{Q})}{dQ}(\tilde{Q} - \tilde{X}_2) = 0$$

and the amount of gas supply is increased. Inasmuch as flaring gas produce carbon dioxide and is a negative environmental externality, charging Pemex for flared gas is a Pigou tax and helps the environment.

Charging Pemex the market price for the gas it consumes will increase the amount supplied if it is facing a smooth demand curve. It does not lead to optimal pricing of gas except in the special case where $(\tilde{Q} - \tilde{X}_1) = 0$. While it is necessary to understand the case where the demand curve is smooth and there are no mass points to understand the

economics of the problem, however, the case that is relevant at the moment is the possible shift of the arbitrage point from Los Ramones to Cempoala. This involves a shift between two mass points connected by a gap in the distribution. The derivative of the demand curve is not defined at Q_c .

IV. Production with Mass Points and Gaps

Continuous case

Let us consider the case where the demand curve is characterized by two mass points connected by a gap in the distribution. Further let us assume that Pemex must pay the market price for the gas it uses or flares. Then Pemex must solve two problems and compare the solutions. First, it must solve the problem where it does not supply any additional gas to the pipeline and pays the penalty for flaring gas. The objective function in this case is

$$(53) \quad \pi = p_c(Q_c - Q_f - X_2) + p_o Z$$

which it maximizes subject to the production constraints:

$$(54) \quad \beta F(X_2) - Q - Q_f - X_2 = 0$$

$$(55) \quad F(X_2) - Z = 0.$$

The Lagrangian is

$$(56) \quad L = p_c(Q_c - Q_f - X_2) + p_o Z + \lambda_1[\beta F(X_2) - Q - Q_f - X_2] + \lambda_2[F(X_2) - Z]$$

Note that Q_c is fixed. The first order conditions for Q_f and Z are:

$$(57) \quad -p_c - \lambda_1 \leq 0 \quad Q_f(p_c + \lambda_1) = 0$$

$$(58) \quad p_o - \lambda_2 \leq 0 \quad Z[p_o - \lambda_2] = 0$$

and since Q_f and Z are strictly positive by assumption, the optimal conditions for the production of gas and oil are

$$(59) \quad \frac{dF(X_2)}{dX_2} = \frac{p_c}{p_c - p_c(\beta + 1)}$$

The alternative is for Pemex not to flare gas and sell an amount of gas greater than Q_a at a price p_a . The maximization problem is then given by

$$(60) \quad \pi = p_a(Q - X_2) + p_o Z$$

which is also maximized subject to the production constraints

$$(61) \quad \beta F(X_2) - Q - X_2 = 0$$

$$(62) \quad F(X_2) - Z = 0$$

The Lagrangian is

$$(63) \quad L = p_a(Q - X_2) + p_o Z + \lambda_1[\beta F(X_2) - Q - X_2] + \lambda_2[F(X_2) - Z].$$

Note that Q_f is assumed to be zero and is not included in the optimization. The first order conditions for Q and Z are:

$$(64) \quad p_a - \lambda_1 \leq 0 \quad Q_f(p_a - \lambda_1) = 0$$

$$(65) \quad p_o - \lambda_2 \leq 0 \quad Z[p_o - \lambda_2] = 0$$

Since Q and Z are strictly positive by assumption, the first-order condition with respect to X_2 is

$$(66) \quad \frac{dF(X_2)}{dX_2} = \frac{p_a}{p_o + p_a(\beta + 1)}$$

Define the net gas produced, X_3 as

$$(67) \quad X_3 = \beta F(X_2) - X_2$$

Equations (62) and (67) can be solved for $Z(X_3)$.

Pemex's profit can be written as a function of X_3 . $\pi(X_3)$, for $X < Q_c$ is given by

$$(68) \quad \pi(X_3) = p_o Z(X_3) + p_c X_3$$

and for $X \geq Q_c$ Pemex's profit is either

$$(69) \quad \pi_1(X_3) = p_o Z(X_3) + p_c Q_c$$

or

$$(70) \quad \pi_2(X_3) = p_o Z(X_3) + p_a X_3$$

So for $X_3 < Q_c$, the price of gas is based on the price at Cempoala, all gas is sold so $Q = X_3$. For $X_3 \geq Q_c$ there are two possibilities. Pemex can either start flaring gas so the income for all $X_3 \geq Q_c$ comes from the sale of oil. Alternatively, Pemex can continue to sell gas, the price is then based on the price at Los Ramones and the profit function is discontinuous at Q_c .

The profit function is illustrated in Figure 5 under the simplifying assumption that Z is proportional to X_3 .

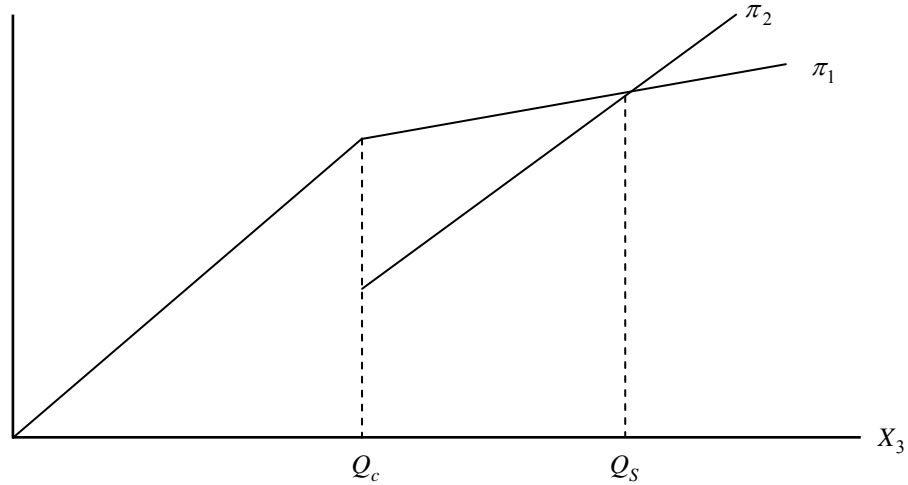


Figure 5

If Pemex supplies more than Q_c gas to the market the point of arbitrage will move from Cempoala to Los Ramones and the price of gas will drop from p_c to p_a . $\pi_1(Q)$ gives Pemex's profit if it does not flare gas and accept the drop in price. $\pi_2(Q)$ gives Pemex's profit if it flares gas and pays the penalty. For quantities of gas less than Q_s it is optimal for Pemex to flare gas.

V. Gas Processing Constraint

Continuous case-Gas is free to Pemex

We have studied the problem under the assumption that gas to be sold to the pipeline without processing. This is not a realistic assumption since it is necessary to remove butane, propane and other natural gas liquids from the natural gas before it can be transmitted in a pipeline. This requires processing and gathering capacity. There is a question whether Pemex has sufficient capacity. The fact that over 200 million cubic feet

of gas are currently flared suggests that this is a problem. Let us assume that Pemex can process \bar{X} amount of gas for sale in the pipeline. Then Pemex would want to maximize the revenue from the sale of oil and gas less the cost of production.

$$(71) \quad \pi = P(Q)Q + p_o Z$$

subject to the production constraints

$$(72) \quad \beta F(X_2) - Q - Q_f - X_2 = 0$$

$$(73) \quad F(X_2) - Z = 0$$

and the gas processing constraint

$$(74) \quad Q \leq \bar{X}$$

where \bar{X} is the constraint on processing capacity. The Lagrangian is

$$(75) \quad L = P(Q)Q + p_o Z + \lambda_1[\beta F(X_2) - Q - Q_f - X_2] \\ + \lambda_2[F(X_2) - Z] + \lambda_3(\bar{X} - Q)$$

where λ_3 is the Lagrange multiplier associated with the gas processing constraint. The first order conditions are:

$$(76) \quad P(Q) + \frac{dP(Q)}{dQ}Q - \lambda_1 - \lambda_3 = 0$$

$$(77) \quad p_o - \lambda_2 = 0$$

$$(78)$$

$$(79) \quad \lambda_1 \geq 0 \quad \lambda_1 Q_f = 0$$

$$(80) \quad \bar{X} - Q \geq 0 \quad \lambda_3(\bar{X} - Q) = 0$$

$$(81) \quad (\lambda_1 \beta + \lambda_2) \frac{dF(X_2)}{dX_2} - \lambda_1 = 0$$

Denote the solution of this problem by a \sim . If $\tilde{Q} \leq \bar{X}$, then there is no change from the previous analysis, so assume that the constraint binds. Then $\tilde{Q} = \bar{X}$ and

$$(82) \quad \tilde{\lambda}_1 + \tilde{\lambda}_3 = P(\bar{X}) + \frac{dP(\bar{X})}{dQ} \bar{X}.$$

There are two cases. The first case is if gas is flared, then $\tilde{Q}_f > 0$, and $\tilde{\lambda}_1 = 0$.

The second case is if gas is not flared. Then $\tilde{Q}_f = 0$ and $\tilde{\lambda}_1 > 0$.

If gas is flared and $\lambda_1 = 0$ then equation (81) can be written as

$$(83) \quad \frac{\partial F(E, X_2)}{\partial X_2} = 0$$

The value of \tilde{X}_2 is determined by equation (83). Further, if gas is being flared $\tilde{\lambda}_1 = 0$ and

$$(84) \quad \tilde{\lambda}_3 = P(\bar{X}) + \frac{dP(\bar{X})}{dQ} \bar{X}.$$

The price of gas is imputed to the gas processing constraint.

If gas is not flared, then all the gas that is not sold will be injected, so $\tilde{X}_2 = X - \bar{X}$. The

values of \tilde{X} , and $\tilde{\lambda}_1$ are determined by the solution of

$$(85) \quad \beta F(E, X - \bar{X}) - X = 0$$

$$(86) \quad (\lambda_1 \beta + p_o) \frac{dF(X - \bar{X})}{dX_2} - \lambda_1 = 0$$

This solution will be used to compare the impact of charging Pemex of the gas it uses.

Continuous case- Pemex pays for Gas

Now let us consider what would happen if Pemex pays the market price for the gas it uses. Pemex would want to maximize the revenue from the sale of oil and gas less the cost of production where the cost of production includes the cost of gas used in the production of gas and oil as well as flared gas.

$$(87) \quad \pi = P(Q)(Q - Q_f - X_2) + p_o Z$$

The Lagrangian is

$$(88) \quad L = P(Q)(Q - Q_f - X_2) + p_o Z + \lambda_1[\beta F(X_2) - Q - Q_f - X_2] \\ + \lambda_2[F(X_2) - Z] + \lambda_3(\bar{X} - Q)$$

If the constraint on the processing of gas for sale in the pipeline is binding, the first order conditions are:

$$(89) \quad P(\bar{X}) + \frac{dP(\bar{X})}{dQ}(\bar{X} - Q_f - X_2) - \lambda_1 - \lambda_3 = 0$$

$$(90) \quad p_o - \lambda = 0$$

$$(91) \quad -P(\bar{X}) - \lambda_1 \leq 0 \quad [P(\bar{X}) + \lambda_1]Q_f = 0$$

$$(92) \quad (\lambda_1\beta + \lambda_2)\frac{dF(X_2)}{dX_2} - P(\bar{X}) - \lambda_1 = 0$$

Denote the solutions by a $\hat{\cdot}$. Let us first consider the case where gas is flared and $\hat{Q}_f > 0$.

then from (91) $\hat{\lambda}_1 = -P(\bar{X})$ so

$$(93) \quad \hat{\lambda}_3 = 2P(\bar{X}) + \frac{dP(\bar{X})}{dQ}(\bar{X} - \hat{X}_2 - \hat{Q}_f) > P(\bar{X}) + \frac{dP(\bar{X})}{dQ}\bar{X} = \tilde{\lambda}_3.$$

The term $\frac{dP(\bar{X})}{dQ} \bar{X} < \frac{dP(\bar{X})}{dQ} (\bar{X} - \hat{X}_2 - \hat{Q}_f)$ so $\hat{\lambda}_3 > \tilde{\lambda}_3$ and the shadow price of gas processing facilities is greater in the case where Pemex must pay for the gas it uses. This result suggests that requiring Pemex to pay for the gas it uses will increase its incentives to invest in gas processing capacity.

If gas is flared, then since, $\lambda_2 = -P(\bar{X})$ and equation (92) be written as

$$(94) \quad \frac{dF(X_2)}{dX_2} = 0$$

Note that equations (83) and (94) are identical so $\hat{X}_2 = \tilde{X}_2$. This gives the somewhat surprising result that if Pemex is flaring gas, having Pemex pay for the gas it uses in production and that it flares does not change any of the short run economic decisions if the gas processing constraint is binding.

Now let us consider the case when Pemex is not flaring gas. The first order condition for the sale of gas results in

$$(95) \quad \lambda_1 + \lambda_3 = P(\bar{X}) + \frac{dP(\bar{X})}{dQ} (\bar{X} - \tilde{X}_2) > P(\bar{X}) + \frac{dP(\bar{X})}{dQ} \bar{X}$$

so $\lambda_1 + \lambda_3$ is greater in the case where Pemex must pay for the gas it uses if Pemex is not flaring gas.

The amount of gas that is used in production is $\hat{X}_2 = \hat{X} - \bar{X}$. Let p be the price of gas to Pemex and define $q = (\lambda_1\beta + p_o)$. The values of \hat{X} , and $\hat{\lambda}_1$ are determined by the solution of

$$(96) \quad \beta F(X_1 - \bar{X}) - X = 0$$

$$(97) \quad q \frac{dF(X_1 - \bar{X})}{dX_2} = p + \frac{q - p_o}{\beta}$$

If we differentiate 95) and 96 with respect to p , we get

$$(98) \quad \begin{pmatrix} \beta \frac{dF}{dX_1} - 1 & 0 \\ q \frac{d^2F}{dX_1^2} & \frac{dF}{dX_1} - \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} \frac{dX_1}{dp} \\ \frac{dq}{dp} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\beta} \end{pmatrix}$$

solving for $\frac{dX_1}{dp}$ we get

$$(99) \quad \frac{dX}{dp} = \frac{\begin{vmatrix} 0 & 0 \\ \frac{1}{\beta} & \frac{dF}{dX_1} - \frac{1}{\beta} \end{vmatrix}}{\begin{vmatrix} \beta \frac{dF}{dX_1} - 1 & 0 \\ q \frac{d^2F}{dX_1^2} & \frac{dF}{dX_1} - \frac{1}{\beta} \end{vmatrix}} = 0$$

and short run production is independent of the price of gas. Solving for

$$(100) \quad \frac{dq}{dp} = \frac{\begin{vmatrix} \beta \frac{dF}{dX_1} - 1 & 0 \\ q \frac{d^2F}{dX_1^2} & \frac{1}{\beta} \end{vmatrix}}{\begin{vmatrix} \beta \frac{dF}{dX_1} - 1 & 0 \\ q \frac{d^2F}{dX_1^2} & \frac{dF}{dX_1} - \frac{1}{\beta} \end{vmatrix}} = \frac{1}{\beta \frac{dF}{dX_1} - 1}$$

and

$$(101) \quad \frac{d\lambda_1}{dp} = \frac{1}{\beta(\beta \frac{dF}{dX_1} - 1)}$$

If Pemex is not flaring gas, it would never be optimal to produce at a point where

$\beta \frac{dF}{dX_1} \geq 1$, since this implies that more gas is being produced than the amount of gas

being used in the production of gas. Therefore making Pemex pay for the gas it uses in production decreases the value of λ_1 . Since the sum $\lambda_1 + \lambda_3$ is larger, it must be that the value of λ_3 is larger. The incentive for Pemex to invest in gas processing capacity is increased.

Mass Points

Now let us consider the case where the distribution is characterized by mass points. The optimization is the same as with a continuous distribution, except that the price of gas is fixed. This is just a special case where $P(Q) = p_k$. However, since this is the case that is most similar to the current situation, it merits a complete treatment. If it is optimal for Pemex to withhold processed gas from market, then the constraint is not binding and the

analysis is as in section IV. Let us assume that the constraint is binding. Then the price Pemex would receive for the gas is given by the price at the mass point, p_k and again Pemex would want to maximize the revenue from the sale of oil and gas less the cost of production. The optimization is given by maximizing

$$(102) \quad \pi = p_k Q + p_o Z$$

subject to the production constraints

$$(103) \quad \beta F(X_2) - Q - Q_f - X_2 = 0$$

$$(104) \quad F(X_2) - Z = 0$$

and the gas processing constraint

$$(105) \quad Q \leq \bar{X}$$

The Lagrangian is

$$(106) \quad L = p_k Q + p_o Z + \lambda_1 [\beta F(X_2) - Q - Q_f - X_2] \\ + \lambda_2 [F(X_2) - Z] + \lambda_3 (\bar{X} - Q)$$

The first order conditions are:

$$(107) \quad p_k - \lambda_1 - \lambda_3 = 0$$

$$(108) \quad p_o - \lambda_2 = 0$$

$$(109) \quad \lambda_1 \geq 0 \quad \lambda_1 Q_f = 0$$

$$(110) \quad \bar{X} - Q \geq 0 \quad \lambda_3 (\bar{X} - Q) = 0$$

$$(111) \quad (\lambda_1 \beta + \lambda_2) \frac{dF(X_2)}{dX_2} - \lambda_1 = 0$$

Again, there are two possible solutions. First if gas is not flared and $Q_f = 0$, then $\lambda_1 > 0$. Second, if gas is flared, then $Q_f > 0$, and $\lambda_1 = 0$. In both cases λ_1 reflects the value of gas and λ_3 the value of the gas processing constraint. Clearly, if gas is being flared, the price of gas is all imputed to the gas processing constraint.

The first order condition for the use of gas in the production of gas and oil is

$$(112) \quad \frac{dF(X_2)}{dX_2} = \frac{\lambda_1}{(\lambda_1\beta + p_o)}$$

We know that if gas is not flared, then all the gas that is not sold must be injected so the value of λ_1 is determined by $X_2 = X - \bar{X}$ and equations (104) and (112). If gas is flared, the $\lambda_2 = 0$ and

$$(113) \quad \frac{dF(X_2)}{dX_2} = 0$$

Now let us consider what would happen if Pemex pays the market price for the gas it uses. Pemex would want to maximize the revenue from the sale of oil and gas less the cost of production.

$$(114) \quad \pi = p_k(Q - Q_f - X_2) + p_o Z$$

The Lagrangian is

$$(115) \quad L = p_k(Q - Q_f - X_2) + p_o Z + \lambda_1[\beta F(X_2) - Q - Q_f - X_2] \\ + \lambda_2[F(X_2) - Z] + \lambda_3(\bar{X} - Q)$$

Assume the constraint on the processing of gas for sale in the pipeline is binding. The first order conditions are:

$$(116) \quad p_k - \lambda_1 - \lambda_3 = 0$$

$$(117) \quad p_o - \lambda_2 = 0$$

$$(118) \quad -p_k - \lambda_1 \leq 0 \quad [p_k + \lambda_1]Q_f = 0$$

$$(119) \quad (\lambda_1\beta + \lambda_2) \frac{dF(X_2)}{dX_2} - P(\bar{X}) - \lambda_1 = 0$$

Let us first consider the case where gas is flared. In that case $Q_f = 0$ implies that

$$\lambda_1 = -p_k \text{ so}$$

$$(120) \quad \lambda_3 = 2p_k$$

The value of λ_3 will increase so requiring Pemex to pay for the gas it uses will increase its incentives to invest in gas processing capacity.

The first order condition for the use of gas in the production of gas and oil is

$$(121) \quad \frac{dF(X_2)}{dX_2} = \frac{p_k + \lambda_1}{(\lambda_1\beta + p_o)}$$

If gas is flared, then from the Kuhn-Tucker condition given by(118), $\lambda_2 = -p_k$ and

$$(122) \quad \frac{dF(X_2)}{dX_2} = 0.$$

Thus having Pemex pay for the gas it uses will be equivalent to a lump sum tax in that it does not change any of the economic decisions if the gas processing constraint is binding and gas is flared. As we would expect, the results do not change from the results in the more general case.

VI. Conclusions and Recommendations

The possibility that Pemex will behave strategically in its short run supply of gas creates some interesting technical and economic problems. The density function that characterizes demand has mass points and gaps. This results in a profit function that is not concave and standard economic analysis must be used with care.

This paper looks at various models that address strategic behavior in the supply of gas. The models increase in complexity and understanding them is useful in developing a well-informed intuition about the problem. The model that most closely resembles the current situation in Mexico is one where:

1. The distribution is characterized by mass points;
2. Pemex uses gas in the production of gas and oil;
3. The constraint in the processing of gas to pipeline quality is binding;
4. Gas is being flared.

That model has three very strong technical results. First, the netback pricing rule leads to discontinuities in Pemex's revenue function. Second, having Pemex pay for the gas it uses and the gas it flares increases the value of the Lagrange multiplier associated with the gas processing constraint. Third, if the gas processing constraint is binding, having Pemex pay for the gas it uses and flares does not change the short run optimal solution for the optimization problem so it will have no impact on short behavior.

The first recommendation that follows from this analysis is that the arbitrage point be fixed by the amount of gas Pemex has the potential to supply in the absence of processing and gathering constraints. This policy is not strictly optimal in that it violates the Little-

Mirrlees Rule, but the distortion is not large. The cost of distortion is less than the cost of moving the necessary gas from between the two arbitrage points in question. The reasoning behind this recommendation is that the discontinuities in Pemex's revenue function create non-convexities in the optimization problem that cannot be addressed by policies that work at the margin.

This is more of a political than an economic problem. In the absence of intuitional constraints on investment by Pemex, it would be economically efficient to invest in processing capacity so as not to flare gas and supply this gas to market. Given that there are intuitional constraints that restrict investment, the question is whether economic and political benefits of supplying gas to central Mexico at the price that would prevail in the absence of these constraints outweighs the cost of transporting gas between the two arbitration points.

The second recommendation that follows from this analysis is that Pemex be charged for the gas it uses in production and the gas it flares. In the short run, this policy is neutral in that it does not distort behavior. In the long run, it creates incentives for Pemex to invest in gas processing capacity.

The third recommendation that is suggested by this study is that investment in gas processing and pipeline be in a separate account from other Pemex investment. Pemex is under a strict capital constraint. The reasons for this constraint are beyond the scope of this paper. However, capacity constraints in gas processing appear to be a serious problem; Mexico is flaring a substantial amount of gas while it is importing gas from the United States. Pipeline capacity is not a binding constraint at the moment. However,

demand is growing. If there is not sufficient investment in pipelines, capacity constraints may become binding.

Finally, a study should be done of the demand elasticity for gas in the production of gas and oil. At the moment Pemex appears to be treating gas as a free good. The question is how much gas would be available if Pemex had to pay for the gas it uses and the gas processing constraint was not binding. This is a question for petroleum and reservoir engineers.

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