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Welfare Evaluation of an Arbitrary Policy Change

The basic analysis of welfare change using CV and EV considers the case of a single price change. However, what should we do if the policy change is not a single price change? For changes in multiple prices, we can just compute the CV for each of the changes (i.e., changing prices one by one and adding the CV (or EV) from each of the changes along a "path" from the original price to the new price). If price and wealth change, we can add the change in wealth to the CV (or EV) from the price changes (see below). But, what if the policy change involves something other than prices and wealth, such a change in environmental quality, roads, etc. How do we value such a change?

The answer is that, if we have good estimates of Walrasian demand, we can always represent the change as a change in a budget set. After doing so, we can compute the CV is the usual way.

Part 1: Any arbitrary policy change can be thought of as a simultaneous change in p and w.

To illustrate, suppose that we have a good estimate of consumers' demand functions (i.e., we fit a flexible functional form for demand using high-quality data). Let $x(p, w)$ denote demand. Suppose that initially prices and wealth are $\left(p^0, w^0\right)$ and the consumer chooses bundle $x\left(p^0, w^0\right)$. Now, suppose that "something happens" that leads the consumer to choose bundle x' instead of x^0. What is the CV (or EV) of this change?

The first step is to note that, if demand is quasiconcave, there is some price-wealth vector for which x^0 and x' are optimal choices. You can find these price-wealth vectors, which we'll call $\left(p^0, w^0\right)$ and (p', w'), by solving the equations $x^0 = \left(p^0, w^0\right)$ and $x' = x(p', w')$. (In reality you probably already know $\left(p^0, w^0\right)$ and have an observation of x' or estimate of.) Remember, we have a good estimate of $x(p, w)$. Once we find (p', w'), then we know that the change in the consumer's utility in going from x^0 to $x^{'}$ is just $v(p', w') - v\left(p^0, w^0\right)$, and so the impact of the policy change reduces to computing the EV or CV for this simultaneous change in p and w. Let $v(p', w') = u'$ and $v\left(p^0, w^0\right) = u^0$.

Part 2: Compute the EV or CV for a simultaneous change in p and w.

So, we've recast the policy change as a change from $\left(p^0, w^0\right)$ to (p', w'), letting u^0 and u' denote the utility levels before and after the change. To compute EV, return to the definition of EV we used before.

$$EV = e\left(p^0, u'\right) - e\left(p^0, u^0\right)$$

Adding and subtracting $e(p', u')$, we get:

$$EV = \left[e\left(\, p^0, u'\right) - e\left(\, p', u'\right)\right] + \left[e\left(p', u'\right) - e\left(p^0, u^0\right)\right].$$

But, note that $e(p', u') = w'$ and $e\left(p^0, u^0\right) = w^0$, so

$$EV = \left[e\left(\, p^0, u'\right) - e\left(\, p', u'\right)\right] + w' - w^0, \qquad (*)$$

and note that $\left[e\left(p^0, u'\right) - e(p', u')\right]$ is as in the definition of EV when only a price changes. So, if only the price of good 1 changes, EV can be written as:

$$EV = \int_{p_1'}^{p_1^0} h_1\left(s, p_{-1}, u'\right) ds + \left(w' - w^0\right),$$

and this can be estimated in the usual way from the estimated Walrasian demand curve.

If multiple prices change, we change them one by one and add up the integral from each change, and then we add the change in wealth. That is, if prices change from $\left(p_1^0, p_2^0, ..., p_L^0\right)$ to $(p_1', p_2', ..., p_L')$ and wealth changes from w^0 to w', the EV is:

$$EV = \int_{p_1'}^{p_1^0} h_1\left(s, p_2^0, ..., p_L^0\right) ds + \int_{p_2'}^{p_2^0} h_2\left(p_1', s, p_3^0, ..., p_L^0\right) ds + ... + \int_{p_L'}^{p_L^0} h_2\left(p_1', p_2', ..., p_{L-1}', s\right) ds + w' - w^0.$$

If you replace each Hicksian demand with an estimate based on Marshallian demand and the Slutsky equation, you can estimate this using only observables. It is tedious, but certainly possible.

This is a diagram that illustrates the whole thing. Suppose a policy change moves the consumer's consumption bundle from x^0 to x'. To compute the EV, the first thing you do is find the (p, w) for which $x^0 = x\left(p^0, w^0\right)$. This budget set is labeled $B\left(p^0, w^0\right)$. Then, you find the (p, w) for which x' is optimal, which we call (p', w'). This budget set (red) is labeled $B(p', w')$. Denote the initial utility level u^0 and the final utility level u', and note that neither the utility levels nor the indifference curves (which are drawn in as dotted lines for illustration) are observed.

Next, we decompose the change from $\left(p^0, w^0\right)$ to (p', w') into two parts. Part 1 is a change in wealth holding prices fixed at p^0. Let y denote the point the consumer chooses at $\left(p^0, w'\right)$, and let u^y denote the utiltiy earned. This point and the associated budget set are in blue. Note that moving from budget set $B\left(p^0, w^0\right)$ to budget set $B\left(p^0, w'\right)$ is just like losing $w' - w^0$ dollars (since prices don't change this is, in fact, exactly what happens). This is where the $\left(w' - w^0\right)$ term comes from in expression (*) above. Distance $w' - w^0$ is denoted on the left. (Note that in the diagram, these distances are scaled by p_2, since we are showing them on the x_2-axis.)

Part 2 of the decomposition is the change in prices from p^0 to p' when wealth is w'. But, note that this just the kind of EV we computed in the simple case. That is, prices change but wealth remains constant. Let z denote the point that offers the same utility as x' but is chosen at prices p'. That is, $z = x\left(p^0, w' + EV\left(p^0, p'.w'\right)\right)$. The budget line supporting z is denoted in green, and the EV for the price change from p^0 to p' at wealth w' is just the distance that the budget shifts up from blue $B\left(p^0, w'\right)$ to green $B\left(p^0, w' + EV\left(p^0, p'.w'\right)\right)$ denoted $EV\left(p^0, p', w'\right)$ on the left. Since

$$\begin{aligned} EV\left(p^0, p', w'\right) &= e\left(p^0, u'\right) - e\left(p^0, u^y\right) \\ &= e\left(p^0, u'\right) - w' \\ &= e\left(p^0, u'\right) - e\left(p', u'\right), \end{aligned}$$

this is just like the EV's we computed when only prices changed. This is where the $\left[e\left(p^0, u'\right) - e\left(p', u'\right)\right]$ comes from in expression (*) above.

The total EV is the sum of these two parts. The distance is denoted Total EV in the diagram. Note that since the consumer ends up worse off overall, the total EV should be negative.

You could also do something similar for CV.

$$\begin{aligned} CV &= e\left(p', u'\right) - e\left(p', u^0\right) \\ &= \left[e\left(p', u'\right) - e\left(p^0, u^0\right)\right] + \left[e\left(p^0, u^0\right) - e\left(p^{'}, u^0\right)\right] \\ &= w' - w^0 + \left[e\left(p^0, u^0\right) - e\left(p^{'}, u^0\right)\right], \end{aligned}$$

and note that once again $\left[e\left(p^0, u^0\right) - e\left(p', u^0\right)\right]$ is as in our original definition of CV. So, this term can be rewritten in terms of integrals of Hicksian demand curves at utility level u^0.

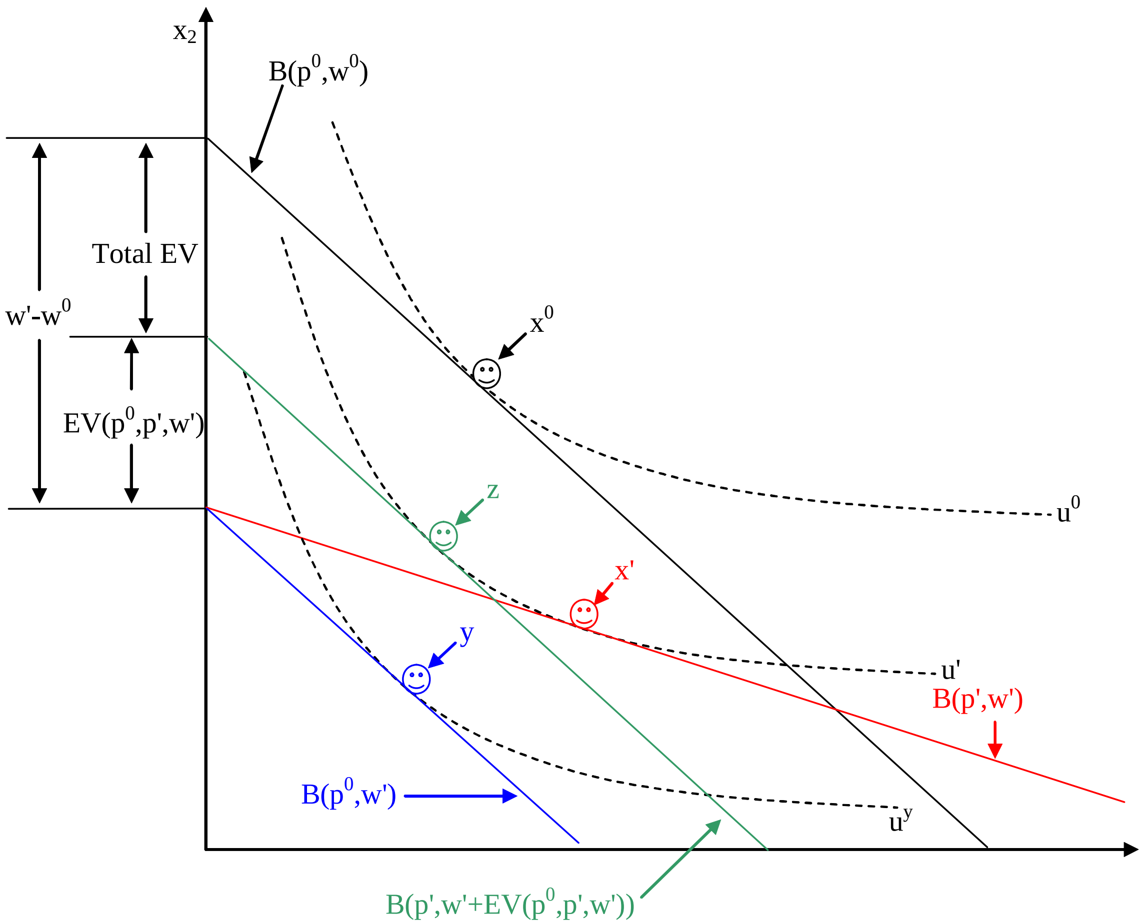

Figure 1: