

## Chapter 7

# Competitive Markets and Partial Equilibrium Analysis

Up until now we have concentrated our efforts on two major topics - consumer theory, which led to the theory of demand, and producer theory, which led to the theory of supply. Next, we will put these two parts together into a market. Specifically, we will begin with competitive markets. The key feature of a competitive market is that producers and consumers are considered **price takers**. That is, individual actors can buy or sell as much of the output as they want at the market price, but no one can take any unilateral action to affect the price. If this is the case, then the actors take prices as exogenous when making their decisions, which was a key feature in our analysis of consumer and producer behavior. Later, when we study monopoly and oligopoly, we will relax the assumption that firms cannot affect prices.

Our main goal here will be to determine how supply and demand interact to determine the way the market allocates society's resources. In particular, we will be concerned with:

1. When does the market allocate resources efficiently?
2. When, if the government wants to implement a specific allocation, can the allocation can be implemented using the market (possibly by rearranging people's initial endowments)?
3. Why does the market sometimes fail to allocate resources efficiently, and what can be done in such cases?

The third question will be the subject of the next chapter, on externalities and public goods. For now, we focus on the first and second questions, which bring us to the first and second fundamental

theorems of welfare economics, respectively.

## 7.1 Competitive Equilibrium

The basic idea in the analysis of competitive equilibrium is the “law of supply and demand.” Utility maximization by individual consumers determines individual demand. Summing over individual consumers determines aggregate demand, and the aggregate demand curve slopes downward. Profit maximization by individual firms determines individual supply, and summing over firms determines aggregate supply, which slopes upward. Adam Smith’s invisible hand acts to bring the market to the point where the two curves cross, i.e. supply equals demand. This point is known as a competitive equilibrium, and it tells us how much of the output will be produced and the price that will be charged for it.

### Notation

We are going to be dealing with many consumers, many producers, and many commodities. To make things clear, I’ll denote which consumer or producer we are talking about with a superscript. For example,  $u^i$  is the utility function of consumer  $i$ ,  $x^i$  is the commodity bundle chosen by consumer  $i$ , and  $y^j$  is the production plan chosen by firm  $j$ . For vectors  $x^i$  and  $y^j$ , I’ll denote the  $l^{\text{th}}$  component with a subscript. Hence  $x^i = (x_1^i, \dots, x_L^i)$ , and  $y^j = (x_1^j, \dots, x_L^j)$ . So  $x_L^j$  refers to consumer  $j$ ’s consumption of good  $L$ . This differs from MWG, which uses double subscripts. But, I think that this is clearer.

### 7.1.1 Allocations and Pareto Optimality

Our formal analysis of competitive markets begins with defining an allocation, and determining what we mean when we say that an allocation is efficient.

Consider an economy consisting of:

1.  $I$  consumers each with utility function  $u^i$
2.  $J$  firms each maximizing its profit
3.  $L$  commodities.

Initially, there are  $w_l \geq 0$  units of commodity  $l$  available. This societal endowment can either be consumed or used to produce other commodities. Because of this, it is most convenient to use the production plan/ net output vector approach to producer theory.<sup>1</sup>

Each firm has production set  $Y_j$  and chooses production plan  $y^j \in Y_j$  in order to maximize profit. Let  $y_l^j$  be the quantity of commodity  $l$  produced by firm  $j$ . Thus if each firm produces net-input vector  $y^j$ , the total amount of good  $l$  available for consumption in the economy is given by

$$w_l + \sum_j y_l^j.$$

The possible outcomes in this economy are called **allocations**. An allocation is a consumption vector  $x^i \in X^i$  for each consumer  $i$ , and a production vector  $y^j \in Y_j$ . An allocation is **feasible** if

$$\sum_i x_l^i \leq w_l + \sum_j y_l^j.$$

for every  $l$ . That is, if total consumption of each commodity is no larger than the total amount of that commodity available.

Again, one of the things we will be most interested in is efficiency. In the context of producer theory, we considered productive efficiency, the question of whether firms choose production plans that are not wasteful. Currently, we are interested not only in productive efficiency but in consumption efficiency as well. That is, we are concerned that, given the availability of commodities in the economy, the commodities are allocated to consumers in such a way that no other arrangement could make everybody better off. The concept of “making everybody better off” is formalized by **Pareto optimality**.

## Pareto Optimality

When an economist talks about efficiency, we refer to situations where no one can be made better off without making some one else worse off. This is the notion of **Pareto optimality**.

Formally, a feasible allocation  $(x, y)$  is **Pareto optimal** if there is no other feasible allocation  $(x', y')$  such that  $u_i(x'_i) \geq u_i(x_i)$  for all  $i$ , with strict inequality for at least one  $i$ . Thus a Pareto optimal allocation is efficient in the sense that there is no other way to reorganize society’s productive facilities in order to make somebody better off without harming somebody else. Notice that we don’t care about producers in this definition of Pareto optimality. This is okay, because

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<sup>1</sup>Recall that in this approach, inputs enter into the production plan as negative elements.

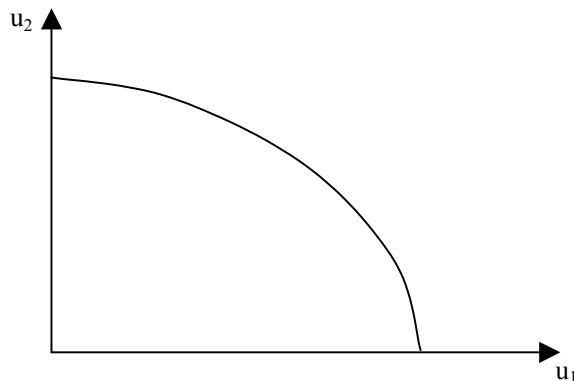


Figure 7.1: Utility Possibility Frontier

all commodities will in the end find their way into the hands of consumers. A profit-maximizing firm will never buy inputs it doesn't use or produce output it doesn't sell, and firms are owned by consumers, so profit eventually becomes consumer wealth. Thus, looking at the utility of consumers fully captures the notion of efficiency.

If you draw the utility possibility frontier in two dimensions, as in Figure 7.1, Pareto optimal points are ones that lay on the northeast frontier. Note that Pareto optimality doesn't say anything about equity. An allocation that gives one person everything and the other nothing may be Pareto optimal. However, it is not at all equitable. Much of the job in policy making is in striking a balance between equity and efficiency – to put it another way, choosing the equitable point from among the efficient points.

### 7.1.2 Competitive Equilibria

We now turn to investigating competitive equilibria with the goal of determining whether or not the allocations determined by the market will be Pareto optimal. Again, we are concerned with competitive markets. Thus buyers and sellers are price takers in the  $L$  commodities. Further, we make the assumption that the firms in the market are owned by the consumers. Thus all profits from operation of the firms are redistributed back to the consumers. Consumers can then use this wealth to increase their consumption. In this way we “close” the model - it's entirely self contained.

Although our formal analysis will be of a partial equilibrium system, where we study only one or two markets, we will define an competitive equilibrium over all  $L$  commodities. In a competitive

economy, a market exists for each of the  $L$  goods, and all consumer and producers act as price takers. As usual, we'll let the vector of the  $L$  commodity prices be given by  $p$ , and suppose consumer  $i$  has endowment  $w_i^i$  of good  $i$ . We'll denote a consumer's entire endowment vector by  $w^i$ , and the total endowment of the good is given by  $\sum_i w_i^i = w_l$ .

We formalize the fact that consumers own the firms by letting  $\theta_j^i$  ( $0 \leq \theta_j^i \leq 1$ ) be the share of firm  $j$  that is owned by consumer  $i$ . Thus if firm  $j$  chooses production plan  $y^j$ , the profit earned by firm  $j$  is  $\pi_j = p \cdot y^j$ , and consumer  $i$ 's share of this profit is given by  $\theta_j^i (p \cdot y^j)$ . Consequently, consumer  $i$ 's total wealth is given by  $p \cdot w_i + \sum \theta_j^i \pi_j$ . Note that this means that all wealth is either in the form of endowment or firm share; there is no longer any exogenous wealth  $w$ . Of course, this depends on firms' decisions, but part of the idea of the equilibrium is that production, consumption, and prices will all be simultaneously determined. We now turn to the formal description of a competitive equilibrium.

There are three requirements for a competitive equilibrium, corresponding to the requirements that producers optimize, consumers optimize, and that "markets clear" at the equilibrium prices. An equilibrium will then consist of a production plan  $y^{j*}$  for each firm, a consumption vector  $x^{i*}$  for each consumer, and a price vector  $p^*$ .

Actually, the producer and consumer parts are just what we have been studying for the first half of the course. The market clearing condition says that at the equilibrium price, it must be that the aggregate supply of each commodity equals the aggregate demand for that commodity, when producers and consumers optimize. Formally, these requirements are:

1. Profit Maximization: For each firm,  $y^j(p)$  solves

$$\max p y^j \text{ subject to } y^j \in Y_j.$$

2. Utility Maximization: For each consumer,  $x^i(p)$  solves

$$\begin{aligned} & \max u^i(x^i) \text{ subject to} \\ & p \cdot x^i \leq p \cdot w^i + \sum \theta_j^i (p \cdot y^j(p)), \end{aligned}$$

where  $\theta_j^i$  is consumer  $i$ 's ownership share in firm  $j$ . Note: this is just the normal UMP with the addition of the idea that the consumer has ultimate claim on the profit of the firm.<sup>2</sup>

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<sup>2</sup>There is something a little strange here. Note that we won't know the firm's profit until after the price vector is determined. But, if we don't know the firm's profit, we can't derive consumers' demand functions, and so we can't

3. Market Clearing. For each good,  $p^*$  is such that

$$\sum_{i=1}^I x_l^i(p^*) = w_l + \sum_{j=1}^J y_l^j(p^*)$$

Of course, we must keep in mind that  $x^*$  and  $y^*$  will be a function of  $p$ . Thus operationally, the requirements for an equilibrium can be written as:

1. For each consumer,  $x^{i*}(p, w^i, \theta^i)$  solves the UMP. Add up the individual demand curves to get aggregate demand,  $D(p)$ , as a function of prices.
2. For each firm,  $y^{j*}(p)$  solves the PMP. Add up the individual supply curves to get aggregate supply,  $S(p)$ , as a function of prices.
3. Find the price where  $D(p^*) = S(p^*)$ .

The last step is the one that you are familiar with from intermediate micro. The first two steps are what we have developed so far in this course. Note that for consumers we will generally need to worry about aggregation issues. However, if consumer preferences take the Gorman form, things will aggregate nicely.

Since  $x^i(p)$  and  $y^j(p)$  are the demand and supply curves, and we know that these functions are homogeneous of degree zero in prices, we know that **if  $p^*$  induces a competitive equilibrium,  $\alpha p^*$  also induces a competitive equilibrium for any  $\alpha > 0$ .** This allows us to normalize the prices without loss of generality, and we will usually do so by setting the price of good 1 equal to 1.

Although we will soon be working with only one or two markets, so far we have been thinking about an economy with  $L$  markets. It can be shown (MWG Lemma 10.B.1) that if you know that  $L - 1$  of the market clear at price  $p^*$ , then the  $L^{\text{th}}$  market must clear as well, provided that consumers satisfy Walras Law and  $p^* \gg 0$ . That is, if

$$\sum_i x_l^i(p^*) = w_l + \sum_j y_l^j(p^*) \text{ for } \forall l \neq k,$$

then

$$\sum_i x_k^i(p^*) = w_k + \sum_j y_k^j(p^*) .$$

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solve the UMP! Actually, this isn't really a problem. The difficulty arises from trying to put a dynamic interpretation on a static model. Really, what we are after is the price which, if it were to come about, would lead to equilibrium behavior. No agent would have any incentive to change what he/she/it is doing. The neoclassical equilibrium model doesn't say anything about how such an equilibrium comes about. Only that if it does, it is stable.

This lemma is a direct consequence of the idea that total wealth must be preserved in the economy. The nice thing about it is that when you are only studying two markets, as we do in the partial equilibrium approach, you know that if one market clears, the other must clear as well. Hence the study of two markets really reduces to the study of one market.

## 7.2 Partial Equilibrium Analysis

### 7.2.1 Set-Up of the Quasilinear Model

We now turn away from the general model to a simple case, known as Partial Equilibrium. It is ‘partial’ because we focus on a small part of the total economy, often on a two commodity world. We laid the groundwork for this type of approach in our discussion of consumer theory. If we are interested in studying a particular market, say the market for apples, we can make the assumption that the prices of all other commodities move in tandem. This justifies, through use of the composite commodity theorem, treating consumers as if they have preferences over apples and “everything else.” Hence we have justified a two-commodity model for this situation. Next, since each consumer’s expenditure on apples is likely to be only a small part of her total wealth, it is reasonable to think of there being no wealth effects on consumers’ demand for apples. And, recall, that quasilinear preferences correspond to the case where there are no wealth effects in the non-numeraire good. So, basically what we’ll do in our partial equilibrium approach (and what is implicitly underlying the approach you took in intermediate micro) is assume that there are two goods: a composite commodity (the numeraire) whose price is set equal to 1, and the good of interest. We’ll call the numeraire  $m$  (for “money”) and the good we are interested in  $x$ .

Now, we can set up the following simple model. Let  $x_i$  and  $m_i$  be consumer  $i$ ’s consumption of the commodity of interest and the numeraire commodity, respectively.<sup>3</sup> Assume that each consumer has quasilinear utility of the form:

$$u^i(m_i, x_i) = m_i + \phi_i(x_i).$$

Further, we normalize  $u^i(0, 0) = \phi_i(0) = 0$ , and assume that  $\phi_i' > 0$  and  $\phi_i'' < 0$  for all  $x_i \geq 0$ . That is, we assume that the consumer’s utility is increasing in the consumption of  $x$  and that her marginal utility of consumption is decreasing.

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<sup>3</sup>This is a change in notation from the set-up at the beginning of the chapter. Now, the subscripts refer to the consumer / firm, rather than the commodity.

Since we already set the price of  $m$  equal to 1, we only need to worry about the price of  $x$ . Denote it by  $p$ .

There are  $J$  firms in the economy. Each firm can transform  $m$  into  $x$  according to cost function  $c_j(q_j)$ , where  $q_j$  is the quantity of  $x$  that firm  $j$  produces, and  $c_j(q_j)$  is the number of units of the numeraire commodity needed to produce  $q_j$  units of  $x$ . Thus, letting  $z_j$  denote firm  $j$ 's use of good  $m$  as an input, its technology set is therefore

$$Y_j = \{(-z_j, q_j) \mid q_j \geq 0 \text{ and } z_j \geq c_j(q_j)\}.$$

That is, you have to spend enough of good  $m$  to produce  $q_j$  units of  $x$ . We will assume that  $c_j(q_j)$  is strictly increasing and convex for all  $j$ .

In order to solve the model, we also need to specify consumers' initial endowments. We assume there is no initial endowment of  $x$ , but that consumer  $i$  has endowment of  $m$  equal to  $w_{mi} > 0$  and the total endowment is  $\sum_i w_{mi} = w_m$ .

### 7.2.2 Analysis of the Quasilinear Model

That completes the set-up of the model. The next step is to analyze it. Recall that in order to find an equilibrium, we need to derive the firms' supply functions, the consumers' demand functions, and find the market-clearing price.

1. **Profit maximization.** Given the equilibrium price  $p^*$ , firm  $j$ 's equilibrium output  $q_j^*$  must maximize

$$\max_{q_j} pq_j - c_j(q_j)$$

which has the necessary and sufficient first-order condition

$$p^* \leq c'_j(q_j^*)$$

with equality if  $q_j^* > 0$ .

2. **Utility Maximization:** Consumer  $i$ 's equilibrium consumption vector  $(x_i^*, m_i^*)$  must maximize

$$\begin{aligned} & \max m_i + \phi_i(x_i) \\ \text{s.t. } & m_i + p^*x_i \leq w_{mi} + \sum \theta_j^i (p^*q_j^* - c_j(q_j^*)) \end{aligned}$$

- We know that the budget constraint must hold with equality. We can substitute it into the objective function for  $m_i$ , which yields:

$$\max \phi_i(x_i) - p^*x_i + \left[ w_{mi} + \sum \theta_j^i (p^*q_j^* - c_j(q_j^*)) \right].$$

The first-order condition is:

$$\phi_i'(x_i^*) \leq p^*$$

which holds with equality if  $x_i^* > 0$ .

3. Market clearing. Remember that lemma that said if one of the markets clears we know that the other one must clear as well? We will use that to formulate a plan of attack here. Basically, we will find a price vector such that aggregate demand for  $x$  equals aggregate supply of  $q$ ,  $\sum_i x_i(p^*) = \sum_j q_j(p^*)$ , i.e. the market for the consumption commodity clears. Then, we'll use the budget equation to compute the equilibrium level of  $m_i$  for each consumer (since the lemma tells us that the market for the numeraire must clear as well). To begin, assume an interior solution to the UMP and PMP for each consumer and firm. Then  $p^*$ ,  $q_j^*$ , and  $x_i^*$  must solve the system of equations

$$\begin{aligned} p^* &= c_j'(q_j(p)) \text{ for all } j \\ p^* &= \phi_i'(x_i(p)) \text{ for all } i \\ \sum_i x_i(p^*) &= \sum_j q_j(p^*) \end{aligned}$$

Notice that the first  $j$  equations determine each firm's supply function. We can then add them to get aggregate supply, the RHS of the third equation. The next  $i$  equations determine each consumer's demand function. We can add them up to get aggregate demand, which is the LHS of the third equation. The third equation is thus the requirement that at the equilibrium price supply equals demand.

Notice that the equilibrium conditions involve neither the initial endowments of the consumers nor their ownership shares. Thus the equilibrium allocation of  $x$  and the price of  $x$  are independent of the initial conditions. This follows directly from the assumption of quasilinear utility. However, since equilibrium allocations of the numeraire are found by using each consumer's budget constraint, the equilibrium allocations of the numeraire will depend on initial endowments and ownership shares.

From a graphical point of view, the partial equilibrium is as follows:

1. For each consumer, derive their Walrasian demand for the consumption good,  $x_i(p)$ . Add across consumers to derive the aggregate demand,  $x(p) = \sum_i x_i(p)$ . Since each demand curve is downward sloping, the aggregate demand curve will be downward sloping. Graphically, this addition is done by adding the demand curves “horizontally” (as in MWG Figure 10.C.1). Since individual demand curves are defined by the relation:

$$p = \phi'_i(q),$$

The price at which each individual’s demand curve intersects the vertical axis is  $\phi'_i(0)$ , and gives that individual’s marginal willingness to pay for the first unit of output. The intercept for the aggregate demand curve is therefore  $\max_i \phi'_i(0)$ . Hence if different consumers have different  $\phi_i()$  functions, not all demand curves will have the same intercept, and the demand curve will become flatter as price decreases.

2. For each firm, derive the supply curve for the consumption good,  $y_j(p)$ . Add across firms to derive the aggregate supply,  $y(p) = \sum_j y_j(p)$ . For each firm, the supply curve is given by:

$$p = c'_j(q_j).$$

Thus each firm’s supply curve is the inverse of its marginal cost curve. Since we have assumed that  $c''_j() \geq 0$ , the supply curve will be upward sloping or flat. Again, addition is done by adding the supply curves horizontally, as in MWG Figure 10.C.2. The intercept of the aggregate supply curve will be the smallest  $c'_j(0)$ . If firms’ cost functions are strictly convex, aggregate supply will be upward sloping.

3. Find the price where supply equals demand: find  $p^*$  such that  $x(p^*) = y(p^*)$ . Since the market clears for good  $l$ , it must also clear for the numeraire. The equilibrium point will be at the price and quantity where the supply and demand curves cross.

At this point, we can talk a bit about the dynamics of how an equilibrium might come about. This is the story that is frequently told in intermediate micro courses, and I should point out that it is just a story. There is nothing in the model which justifies this approach since we have said nothing at all about how markets will behave if they are out of equilibrium. Nevertheless, I’ll tell the story.

Suppose  $p^*$  is the equilibrium price of  $x$ , but that currently the price is equal to  $p^+ > p^*$ . At this price, aggregate demand is less than aggregate supply:  $D(p^+) < S(p^+)$ . Because of this, there

is a “glut” on the market. There are more units of  $x$  available for sale than people willing to buy (think about cars sitting on a car lot at the end of the model year). Hence (so the story goes) there will be downward pressure on the price as suppliers lower their price in order to induce people to buy. As the price declines, supply decreases and demand increases until we reach equilibrium at  $p = p^*$ . Similarly, if initially the price  $p^-$  is such that  $p^- < p^*$ , then  $D(p^-) > S(p^-)$ . There is excess demand (think about the hot toy of the holiday season). The excess demand bids up the price as people fight to get one of the scarce units of  $x$ , and as the price rises, supply increases and demand decreases until equilibrium is reached, once again, at  $p = p^*$ .

Thus we have the “invisible hand” of the market working to bring the market into equilibrium. However, let me emphasize once again that stories such as these are not part of our model.

### 7.2.3 A Bit on Social Cost and Benefit

The firm’s supply function is  $y_j(p)$  and satisfies  $p = c'_j(y_j(p))$ . Thus at any particular price, firms choose their quantities so that the marginal cost of producing an additional unit of production is exactly equal to the price. Similarly, the consumer’s demand function is  $x_i(p)$  such that  $p = \phi'_i(x_i(p))$ . Thus at any price, consumers choose quantities so that the marginal benefit of consuming an additional unit of  $x$  is exactly equal to its price.<sup>4</sup> When both firms and consumers do this, we get that, at equilibrium, the marginal cost of producing an additional unit of  $x$  is exactly equal to the marginal utility of consuming an additional unit of  $x$ . This is true both individually and in the aggregate. Thus at the equilibrium price, all units where the marginal social cost is less than or equal to the marginal social benefit are produced and consumed, and no other units are. Thus the market acts to produce an efficient allocation. We’ll see more about this in a little while, but I wanted to suggest where we are going before we take a moment to talk about a few other things.

### 7.2.4 Comparative Statics

As usual, one of the things we will be interested in determining in the partial equilibrium model is how the endogenous parameters of the model vary with changes in the environment. For example, suppose that a consumer’s utility function depends on a vector of exogenous parameters,  $\phi_i(x_i, \alpha)$ ,

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<sup>4</sup>This is true as long as we assume that for any level of output, consumers with the highest willingness to pay (i.e., marginal benefit) are the ones that are given the units of output to consume, which is a reasonable assumption in many circumstances.

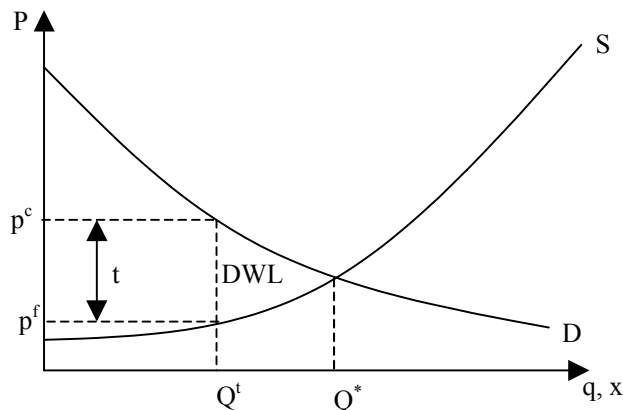


Figure 7.2: Partial Equilibrium with a Tax

and a firm's cost function depends on another vector of parameters (possibly overlapping),  $c_j(q_j, \beta)$ . Note that  $\alpha$  and  $\beta$  will include at least the prices of the commodities, but may include other things such as tax rates, taste parameters, etc. We can ask how the equilibrium prices and quantities change with a change in  $\alpha$  or  $\beta$ .

One of the most studied situations of this type is the impact of a tax on good  $x$ . Suppose, for example, that the government collects a tax of  $t$  on each unit of output purchased. Let the price consumers pay be  $p_c$  and the price producers receive be  $p_f$ . Note that consumers only care about the price they have to pay to acquire the good, and producers only care about the price they receive when they sell a unit of the good. In particular, neither side of the market cares directly about the tax. The two prices are related by the tax rate:

$$p_c = p_f + t.$$

The equilibrium in this market is the point where supply equals demand, i.e., prices  $p_c$  and  $p_f$  such that

$$x(p_c) = q(p_f), \text{ and}$$

$$p_c = p_f + t.$$

Or,

$$x(p_f + t) = q(p_f).$$

The equilibrium is depicted in Figure 7.2:

$Q^t$  is the quantity sold after the tax is implemented. At this quantity, the difference between the price paid by consumers and the price received by firms is exactly equal to the tax. Note that at  $Q^t$ , the marginal social cost of an additional unit of output is less than the marginal social benefit. Hence society could be made better off if additional units of output were produced and sold, all the way up to the point where  $Q^*$  units of output are produced. The loss suffered by society due to the fact that these units are not produced and consumed is called the deadweight loss (DWL) of taxation.

One question we may be interested in is how the price paid by consumers changes when the size of the tax increases. Let  $p^*(t)$  be the equilibrium price received by firms. Thus consumers pay  $p^*(t) + t$ . The following identity holds for any tax rate  $t$ :

$$x(p^*(t) + t) \equiv q(p^*(t)).$$

Totally differentiating with respect to  $t$  yields:

$$\begin{aligned} x'(p^* + t)(p'(t) + 1) &= q'(p^*)p'(t) \\ p'(t) &= \frac{-x'(p^* + t)}{x'(p^* + t) - q'(p^*)} \end{aligned}$$

The numerator is positive by definition. Since  $q'(p^*)$  is positive, the absolute value of the denominator is larger than the absolute value of the numerator, hence  $-1 < p'(t) < 0$ . This implies that as the tax rate increases, the price received by firms decreases, but by less than the full amount of the tax. As a consequence, the price paid by consumers must also increase, but by less than the increase in the tax. Further, the total quantity must decrease as well.

Consider the formula we just derived:

$$p'(t) = \frac{-x'(p^* + t)}{x'(p^* + t) - q'(p^*)}.$$

Evaluate at  $t = 0$ , and rewrite this in terms of elasticities:

$$\begin{aligned} \frac{dp}{dt} &= \frac{-\frac{dx}{dp} \frac{p}{x}}{\frac{dx}{dp} \frac{p}{x} - \frac{dq}{dp} \frac{p}{q}} \\ &= -\frac{|\varepsilon_d|}{|\varepsilon_d| + \varepsilon_s} \end{aligned}$$

where  $\varepsilon_d$  is the elasticity of demand and  $\varepsilon_s$  is the elasticity of supply. This says that the proportion of a small tax that is passed onto producers in the form of lower prices is given by  $\frac{|\varepsilon_d|}{|\varepsilon_d| + \varepsilon_s}$ . The proportion passed onto consumers in the form of higher prices is given by  $\frac{\varepsilon_s}{|\varepsilon_d| + \varepsilon_s}$ .

Now, suppose the government is considering two different tax programs: one that taxes a commodity with relatively inelastic demand, and one that taxes a commodity with relatively elastic demand. Which will result in the larger deadweight loss? The answer is, all else being equal, the commodity with the more elastic demand will have a larger deadweight loss. Why? The more elastic demand is, the flatter the demand curve will be. This means that a more elastic demand curve will respond to a tax with a relatively larger decrease in quantity. And, since this quantity distortion is the source of the deadweight loss, the more elastic demand curve will result in the larger deadweight loss.

Does this mean that we should only tax inelastic things, since this will make society better off? Not really. The main reason is that even though taxing inelastic things may be better for society as a whole from an efficiency standpoint, it may have undesirable redistributive effects. For example, we could tax cigarette smokers and force them to pay for road construction and schools. Since cigarette demand is relatively inelastic, this would result in a relatively small deadweight loss. However, is it really fair to force smokers to pay for roads and schools, even though they don't necessarily use the roads and schools any more intensely than other people? Probably not. We recently had a related issue in Massachusetts. The state wanted to increase tolls on the turnpike in order to pay for construction at the airport. Turnpike usage is relatively inelastic, but is it fair to make turnpike users pay for airport construction, even though turnpike users are no more likely to be going to the airport than other drivers? Issues of balancing efficiency and equity concerns such as this arise often in policy decisions.

### 7.3 The Fundamental Welfare Theorems

Recall that we ended our discussion of production by talking about efficiency, and showed that any profit-maximizing production plan is efficient (i.e. the same output cannot be produced using fewer inputs), and that any efficient production plan (under certain circumstances) is the profit maximizing production plan for some price vector. We now turn to ask the same questions of markets. That is, when are the allocations made by markets "efficient," and is every efficient allocation the market allocation for some initial conditions? Again, the reason we ask these questions has to do with decentralization. When can we decentralize the decisions we make in our society? Do we know that profit-maximizing firms and utility maximizing agents will arrive at a Pareto optimal allocation through the market? If we have a particular Pareto optimal allocation

in mind, can we rely on the market to get us there, provided we start at the right place (i.e., initial endowment for consumers)?

The proper concept of efficiency here is Pareto optimality. Recall that an allocation is Pareto optimal if there is no other feasible allocation that makes all agents at least as well off and some agent better off. We will study Pareto optimal allocations in the context of the quasilinear partial equilibrium model we introduced earlier. This greatly simplifies the analysis, since when preferences are quasilinear, the frontier of the utility possibility set is linear. That is, all points that are Pareto efficient involve the same consumption of the non-numeraire good by the consumers, and differ only in the distribution of the numeraire among the consumers.

To illustrate this point, suppose there are two consumers, and fix the consumption and production levels at  $\bar{x}$  and  $\bar{q}$  respectively. This will leave  $w_m - \sum_j c_j(\bar{q}_j)$  of the numeraire to be distributed among the consumers. Since the numeraire can be traded one-for-one among consumers, the utility possibility set for this  $x^*$  and  $q^*$  is the set

$$\left\{ (u_1, u_2) \mid u_1 + u_2 \leq \phi_1(\bar{x}_1) + \phi_2(\bar{x}_2) + w_m - \sum_j c_j(\bar{q}_j) \right\}$$

This is the utility possibility set for any particular allocation of the consumption good,  $(\bar{x}, \bar{q})$ , if we allow the remaining numeraire to be distributed among consumers in any possible way. The utility possibility set for the efficient allocation is the set generated by the  $x_1^*$  and  $x_2^*$  that maximize the right hand side of this expression: That is, Pareto optimal allocations satisfy:

$$\begin{aligned} x_1^*, x_2^*, q_j^* &\in \arg \max \phi_1(x_1) + \phi_2(x_2) + w_m - \sum_j c_j(q_j) \\ \text{subject to} & : x_1 + x_2 = \sum q_j \end{aligned}$$

We will call the  $x$ 's and  $q$ 's generated by such a procedure the optimal production and consumption levels of good  $x$ . If the firms have strictly concave production functions and  $\phi_i(\cdot)$  is strictly concave, then there will be a unique  $(x, q)$  that maximizes the above expression.

We can rewrite the above problem in the multiple consumer case as:

$$\begin{aligned} \max_{x, q} & \left( \sum_i \phi_i(x_i) \right) + w_m - \left( \sum_j c_j(q_j) \right) \\ \text{subject to} & : \sum_{i=1}^I x_i - \sum_{j=1}^J q_j = 0 \end{aligned}$$

The top line just says to maximize the sum of the consumers' utilities. The constraint is that the total consumption of  $x$  is the same as the total production. Letting the Lagrange multiplier be  $\mu$ , the first-order conditions for this problem are:

$$\begin{aligned}\phi'_i(x_i^*) &\leq \mu \text{ with equality if } x_i^* > 0 \\ c'_j(q_j^*) &\geq \mu \text{ with equality if } q_j^* > 0 \\ \sum_{i=1}^I x_i^* &= \sum_{j=1}^J q_j^*\end{aligned}$$

Note that these are exactly the conditions as the conditions defining the competitive equilibrium except that  $p^*$  has been replaced by  $\mu$ . In other words, we know that the allocation produced by the competitive market satisfies these conditions, and that  $\mu = p^*$ . Thus the competitive market allocation is Pareto optimal, and the market clearing price  $p^*$  is the shadow value of the constraint: the additional social benefit generated by consuming one more unit of output or producing one less unit of output. Hence this is just another expression of the fact that at  $p^*$  the marginal social benefit of additional output equals the marginal social cost.

The preceding argument establishes **the first fundamental theorem of welfare economics** in the partial equilibrium case. *If the price  $p^*$  and the allocation  $(x^*, q^*)$  constitute a competitive equilibrium, then this allocation is Pareto optimal.*

The first theorem is just a formal expression of Adam Smith's invisible hand – the market acts to allocate commodities in a Pareto optimal manner. Since  $p^* = \mu$ , which is the shadow price of additional units of  $x$ , each firm acting in order to maximize its own profits chooses the output that equates the marginal cost of its production to the marginal social benefit, and each consumer, in choosing the quantity to consume in order to maximize utility, is also setting marginal benefit equal to the marginal social cost.

Note that while this is a special case, the first welfare theorem will hold quite generally whenever there are complete markets, no matter how many commodities there are. It will fail, however, when there are commodities (things that affect utility) that have no markets (as in the externalities problem we'll look at soon).

As we did with production, we can also look at this problem “backward.” Can any Pareto optimal allocation be generated as the outcome of a competitive market, for some suitable initial endowment vector? The answer to this question is yes.

To see why, recall that when all  $\phi_i()$ 's are strictly concave and all  $c_j()$ 's are strictly convex, there is a unique allocation of the consumption commodity  $x$  that maximizes the sum of the consumers'

utilities. The set of Pareto optimal allocations is derived by allocating the consumption commodity in this manner and varying the amount of the numeraire commodity given to each of the consumers. Thus the set of Pareto optimal allocations is a line with normal vector  $(1,1,1,\dots,1)$  (see, for example, Figure 10.D.1 in MWG), since one unit of utility can be transferred from one consumer to another by transferring a unit of the numeraire.

Thus any Pareto optimal allocation can be generated by letting the market work and then appropriately transferring the numeraire. But, recall that firms' production decisions and consumers' consumption decisions do not depend on the initial endowment of the numeraire. Because of this, we could also perform these transfers before the market works. This allows us to implement any point along the Pareto frontier.

To see why, let  $(x^*, q^*)$  be the Pareto optimal allocation of the consumption commodity, and suppose we want to implement the point where each consumer gets  $(x_i^*, m_i^*)$  after the market works, where  $\sum_i m_i^* = w_m - \sum_j c_j(q_j^*)$ . If we want consumer  $i$  to have  $m_i^*$  units of the numeraire after the transfer, we need him to have  $m_i^0$  before the transfer, where

$$m_i^0 + \sum_j \theta_j^i (p^* q_j^* - c_j(q_j^*)) = m_i^* + p^* x_i^*.$$

Hence if people have wealth  $m_i^0$  before the market starts to work, allocation  $(x^*, q^*, m^*)$  will result.

This yields the second fundamental theorem of welfare economics. Let  $u_i^*$  be the utility in a Pareto optimal allocation for some initial endowment vector. There exists a set of transfers  $T_i$  (the amount of the numeraire given to consumer  $i$ ) such that  $\sum_i T_i = 0$  and the allocation generated by the competitive market yields the utility vector  $u^*$ .

The transfers are given by  $T_i = m_i^0 - w_{mi}$ .

### 7.3.1 Welfare Analysis and Partial Equilibrium

Recall in our discussion of consumer theory we said that equivalent variation is the proper measure of the impact of a policy change on consumers, and that EV is given by the area to the left of the Hicksian demand curve between the initial and final prices. However, since there are no wealth effects for the consumption good here, we know that the Hicksian and Walrasian demand curves are the same. So, the area to the left of the Walrasian demand curve is a proper measure of consumer welfare. Further, since utility is quasilinear, it makes sense to look at aggregate demand, and there is a normative representative consumer whose preferences are captured by the aggregate demand curve. Hence the area to the left of the Walrasian demand curve is a good measure of changes in

social welfare.

It is worthwhile to derive a measure of aggregate social surplus here, even though we already did it during our study of aggregation. Recall the Pareto optimality problem:<sup>5</sup>

$$\begin{aligned} & \max_{x,q} \left( \sum_i \phi_i(x_i) \right) + w_m - \left( \sum_j c_j(q_j) \right) \\ \text{subject to} \quad & : \quad \sum_{i=1}^I x_i - \sum_{j=1}^J q_j = 0. \end{aligned}$$

We said that the solution to this problem determines the allocation that maximizes consumers' welfare (and therefore societal welfare).

Consider the objective function:

$$\left( \sum_i \phi_i(x_i) \right) - \left( \sum_j c_j(q_j) \right) + w_m.$$

The last term is the initial aggregate endowment of the numeraire good, which is just a constant in the objective function. The first two terms represent the difference between aggregate utility from consumption and aggregate cost of production. It is this difference that we are maximizing in the Pareto optimality problem.

Consider a single unit of production. The difference between the utility derived from that production and the cost of production is the societal benefit from that good (since all profits return to consumers). If we add this surplus across all consumers, that gets us

$$\left( \sum_i \phi_i(x_i) \right) - \left( \sum_j c_j(q_j) \right)$$

which is the surplus generated by all units of production. This term, called Marshallian Aggregate Surplus (MAS), is the measure of social benefit that we want to use, since it tells us how much better off society is made when  $\sum_i x_i = \sum_j q_j$  units of the non-numeraire good are produced and sold.

To better understand, note that we can break the surplus down into four parts:

1. (a) Some of the surplus comes from consumption,  $\sum_i \phi_i(x_i)$ ,
- (b) Some surplus is lost due to paying price  $p$  for the good.
- $\sum_i \phi_i(x_i) - px_i$  is the aggregate consumer surplus

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<sup>5</sup>This corresponds to the utilitarian social welfare problem.

2. (a) Some surplus is gained by firms in the form of revenue,  $\sum_j pq_j$
  - (b) Some surplus is lost by firms in the form of production cost,  $\sum_j c_j(q_j)$
- $\sum_j pq_j - c_j(q_j)$  is the aggregate producer surplus, which is then redistributed to consumers in the form of dividends,  $\theta_{ij}(pq_j - c_j(q_j))$ .

So, consumers receive part of the benefit through consumption of the non-numeraire good,

$$\sum_i \phi_i(x_i) - px_i$$

and part of the benefit through consumption of the dividends, which are measured in units of the numeraire:

$$\sum_j pq_j - c_j(q_j).$$

Aggregate surplus is found by adding these two together, and noting that  $\sum_i px_i = \sum_j pq_j$

In our quasilinear model, the set of utility vectors that can be achieved by a feasible allocation is given by:

$$\left\{ (u_1, \dots, u_I) \mid \sum_i u_i \leq w_m + \left( \sum_i \phi_i(x_i) \right) - \left( \sum_j c_j(q_j) \right) \right\}$$

Suppose that the government (or you, or anybody) has a view of society that says the total welfare in society is given by:

$$W(u_1, \dots, u_I)$$

Thus this function gives a level of welfare associated with any utility vector – it allows us to compare any two distributions of utility in terms of their overall social welfare. The problem of the social planner would be to choose  $u$  in order to maximize  $W(u)$  subject to the constraint that  $u$  lies in the utility possibility set. Clearly, then, the optimized level of  $W$  will be higher when the utility possibility set is larger, which occurs when the MAS is larger. Thus the total societal welfare achievable is increasing in the MAS. See MWG Figure 10.E.1.

In other words, if you want to maximize social welfare, you should first choose the production and consumption vectors that maximize MAS, and then redistribute the numeraire in order to maximize the welfare function. This gives us another separation result: If utility can be perfectly transferred between consumers (as in the quasilinear model), then social welfare is maximized by first choosing production and consumption plans that maximize MAS, and then choosing transfers such that  $u_i = \phi_i(x_i) + m_i + t_i$  maximizes  $W(u)$ .

Now, how does MAS change when the quantity produced and consumed changes? Let  $S(x, q)$  be the MAS, formally defined as follows:

$$S(x, q) = \left( \sum_i \phi_i(x_i) \right) - \left( \sum_j c_j(q_j) \right).$$

Consider a differential increase in consumption and production:  $(dx_1, dx_2, \dots, dx_I, dq_1, \dots, dq_J)$  satisfying  $\sum_i dx_i = \sum_j dq_j$ . Note that under such a change, we increase total production and total consumption by the same amount.

The differential in S is given by

$$dS = \left( \sum_i \phi'_i(x_i) dx_i \right) - \left( \sum_j c'_j(q_j) dq_j \right).$$

Since consumers maximize utility,  $\phi'_i(x_i) = p(x)$  for all  $i$ , and since producers maximize profit,  $c'_j(q_j) = c'(q)$  for all  $j$ . Thus:

$$dS = \left( p(x) \sum_i dx_i \right) - \left( c'(q) \sum_j dq_j \right)$$

which by definition of our changes (and market clearing) implies

$$dS = (p(x) - c'(q)) dx$$

And, integrating this from 0 to  $\bar{x}$  yields that MAS equals

$$S(x) = \int_0^x (p(s) - c'(s)) ds.$$

Thus the total surplus is the area between the supply and demand curves between 0 and the quantity sold,  $\bar{x}$ .

### Example: Welfare Effects of a Tax

We return to the idea of a commodity tax that we first considered in the context of consumer theory. Suppose there is a government that attempts to maximize the welfare of its citizens. The government keeps a balanced budget, and tax revenues are returned to consumers in the form of a lump sum transfer.

What are the welfare effects of this tax? Define  $x_1^*(t), \dots, x_I^*(t)$  and  $q_1^*(t), \dots, q_J^*(t)$  and  $p^*(t)$ , respectively, to be the consumptions, productions, and price paid by consumers when the per-unit

tax is  $t$ . Define  $x^*(t) = \sum x_i^*(t)$  and  $q^*(t) = \sum q_j^*(t)$  to be aggregate consumption and production, respectively. Letting  $S^*(t) = \sum x_i^*(t) - \sum q_j^*(t)$  be the level of MAS (also equal to the area below the demand curve and above the supply curve) at tax rate  $t$ , the change in MAS when a tax of  $t$  is imposed is given by:

$$\begin{aligned} S^*(x^*(t)) - S^*(x^*(0)) &= x^*(t) - q^*(t) - (x^*(0) - q^*(0)) \\ &= \int_{x^*(0)}^{x^*(t)} p(s) - c'(s) ds, \end{aligned}$$

by the definition of MAS developed earlier. Thus the change in MAS is given by the change in the area between the aggregate demand and supply curves, between the equilibrium quantity when there is no tax and the equilibrium quantity after the tax is imposed. Note that this is just the area we called deadweight loss earlier.

## 7.4 Entry and Long-Run Competitive Equilibrium

Up until now, we have considered the competitive equilibrium holding the supply side of the market fixed. In particular, we have assumed:

1. Firms are unable to vary their fixed factors of production (plant size is fixed)
2. Firms are unable to enter or exit the market – the number of firms stay fixed

These assumptions are appropriate in the short run. However, if we want to examine the behavior of the market in the long run, we must explicitly allow for firms to change their fixed factors, including entering or exiting the industry.

In the long run, a perfectly competitive market is characterized by:

1. Firms and consumers are price takers
2. Free entry and exit

The free entry and exit condition doesn't mean that firms can enter at no cost. Rather, it means that there is no impediment to them incurring the cost and entering the market. For example, there are no laws against entry, there are no proprietary technologies or scarce resources, etc. Thus firms have the freedom to enter or exit, but this is not to say that they can do it for free. In our discussion up until now, we have considered only the price-taking requirement. In order to think about competitive equilibrium in the long run, we add the free entry condition.

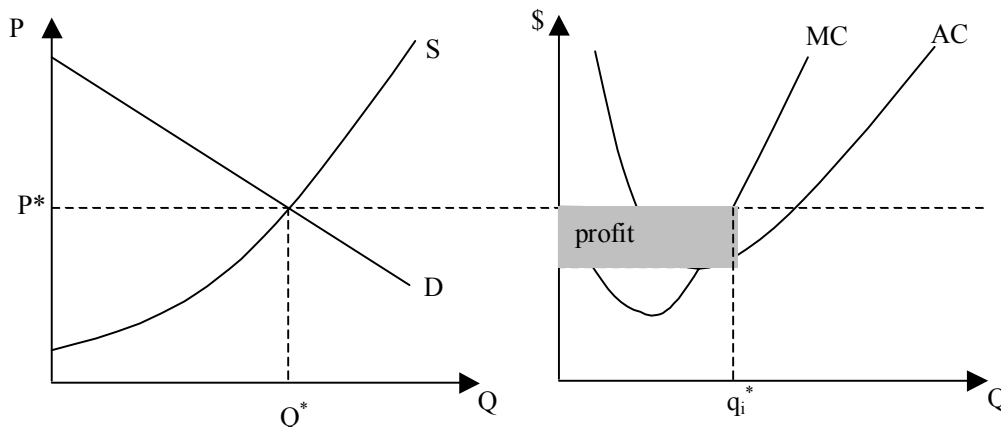


Figure 7.3: Short-Run Equilibrium

### 7.4.1 Long-Run Competitive Equilibrium

Consider our analysis of perfect equilibrium in the short run.<sup>6</sup> The short-run equilibrium price and quantity are found where supply equals demand for a given number of firms. However, note that if there happen to be a small number of firms in an industry, it may be that the firms are making large profits.

Figure 7.3 links the market equilibrium with the individual firm. On the left, aggregate supply and demand combine to determine the equilibrium price,  $P^*$ . At this price, the individual firm's behavior is depicted in the right panel. The firm chooses to produce  $q_i^*$  units of output, and earns  $q_i^*(P^* - AC(q_i^*))$  profit. Total profit by the firm is shaded in the right-hand panel.

Now, if you are a business person, you see an industry where firms are making large profits, and you can enter it if you want, what do you do? You enter. When you enter, what happens to the supply curve? It shifts out to the right. And, as the supply curve shifts right, the equilibrium price decreases, which decreases the profit of the firms already in the industry.

How many firms enter? As long as a firm can earn a positive profit by entering the industry, it will choose to enter. Thus firms will continue to enter until they drive profit to zero. This situation is shown in Figure 7.4.

Note that profit equals zero when the equilibrium price, denoted by  $P'$  in the diagram, is such

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<sup>6</sup>Implicit in the arguments for this section is the idea that all firms, both those that are in the market and those who could potentially enter the market, have the same technology. The conclusions can be adapted to the case where technologies are heterogeneous without changing the results too much.

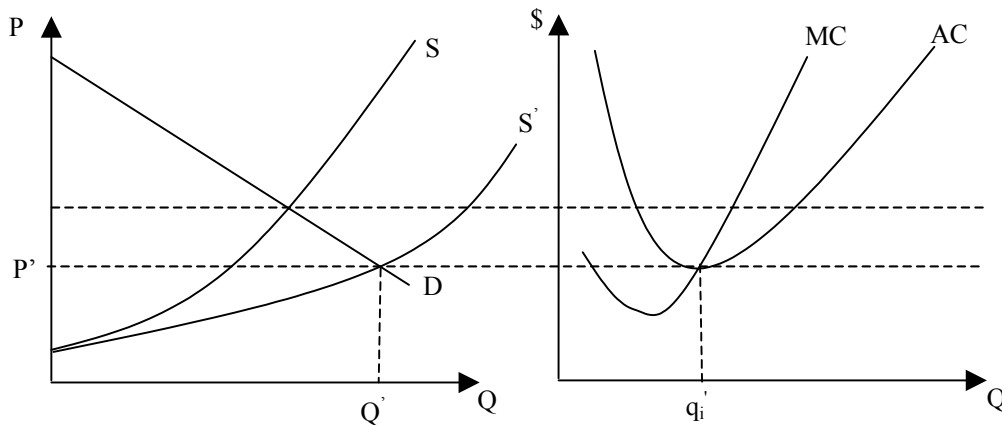


Figure 7.4: Long-Run Equilibrium

that  $P' = \min AC$ .

Similarly, if there are initially too many firms in the industry, the firms in the industry will earn negative profits. This will drive firms to exit the industry. As they exit the industry, the price will increase, and the size of the firms' losses will decrease. When does exit stop? When the firms that are in the industry earn zero profit. You should draw the diagram for this case.

The dynamic story I've been telling suggests the following requirements for a long-run competitive equilibrium. First, we keep the requirements for a short-run equilibrium:

1. (a) Firms maximize profits
- (b) Consumers maximize utility
- (c) Market clearing: price adjusts so that supply equals demand

Second, we add the additional requirement that the equilibrium number of firms is found where in the short-run equilibrium firms make zero profit.<sup>7</sup>

Formally, if there are a very large number of identical firms that could potentially enter this market, this means that the equilibrium consists of  $q^*, p^*, J^*$  that satisfy:

1.  $q^*$  maximizes  $p^*q - c(q)$  for each firm.
2.  $x_i^*$  maximizes  $u_i(x_i) - px_i$  for all  $i$

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<sup>7</sup>There is something of an integer problem here. That is, it may be if there are  $J^*$  firms all firms earn a positive profit, but if there are  $J^* + 1$  firms, all firms earn a negative profit. In this case, we will say that the equilibrium is the largest number of firms such that firms do not earn negative profits.

3.  $x(p^*) = J^*q^*$  : market clearing
4.  $p^*q^* - c(q^*) = 0$  : free entry

The last requirement is one way to think of the free entry condition. Entry continues until all firms make zero profit. Another way to think of it is as follows: Entry will continue until the point that with  $J^*$  firms in the industry, all firms make non-negative profits. With  $J^* + 1$  firms in the industry, all firms make negative profits. Thus it need not be the case that all firms make zero profits. But, it must be the case that if one more firm enters, all firms will make negative profits. Thus  $J^*$  is the maximum number of firms that can be supported by this market.

What will the long-run aggregate supply correspondence look like? Since all firms are the same, the long-run aggregate supply correspondence as a function of  $p$  will look as follows:

$$\begin{aligned} Q(p) &= \infty \text{ if } \pi(p) > 0 \\ &= Jq \text{ for some integer } J \geq 0 \text{ and } q \in q(p) \text{ if } \pi(p) = 0 \\ &= 0 \text{ if } \pi(p) < 0 \end{aligned}$$

That is, if the price is such that profits are positive, an infinite number of firms will enter the industry – driving the quantity supplied to infinity. If price is such that profits are zero then some integer number of firms will enter the market and produce  $q$  according to its supply function  $q(p)$ . When profits are negative at a specific price, firms will supply nothing. Generally, however, we won't worry about the integer problems here. We'll assume that in the long run, the supply curve is horizontal at the level of minimum average cost.

**Example: Consider the case of constant returns to scale technology with no fixed cost:**

$$c(q) = cq.$$

- In this case, firms will supply an infinite amount when  $p > c$ , any positive amount when  $p = c$ , and zero when  $p < c$ .
- The long-run equilibrium will be where demand and long-run supply cross, which is at  $p = c$ . However, we don't know how many firms there will be, since the firms could split up the quantity they want to produce in any way they want. It is a long-run equilibrium with any number of firms!

**Example: Firms have strictly increasing, strictly convex cost**

- In this case, firms make a positive profit whenever  $p > c'(0)$ . Hence entry will continue, driving price down until it reaches  $c'(0)$ , and all firms make zero profit.
- This makes sense – convex cost is the same as decreasing returns to scale. In this case, the most efficient firms are those that are producing no output at all. So, as price decreases, firms are driven to produce more efficiently, and that involves producing lower quantities.

**Example: Firms Have U-Shaped Cost Curves**

Now suppose that the firm has a cost function that is first concave, and then convex, giving a U-shaped average cost curve. In the long run, price will be driven down to the point where  $p = \min AC$ . In that case, each firm will produce the quantity that minimizes  $AC$ . Thus the long-run aggregate supply function is given by:

$$\begin{aligned} Q(p) &= \infty \text{ if } p > \min AC \\ &= J\bar{q} \text{ if } p = \min AC, \text{ where } \bar{q} = q(\min AC) \\ &= 0 \text{ if } p < \min AC \end{aligned}$$

Note that it is possible that when  $p = \min AC$ , there is no  $J$  such that  $J \cdot q(\min AC) = x(\min AC)$ . This is known as the integer problem. There are several reasons to think that the integer problem is not such a horrible thing:

- If firms are small relative to the market, then there will be a  $J$  such that  $J \cdot q(\min AC)$  is close to  $x(\min AC)$
- Long-run equilibrium is a theoretical construct – we never really get there, but we're always moving toward there.
- If the price is slightly above  $\min AC$ , then the supply curve will be upward sloping. Thus we are really looking for the largest  $J$  such that  $p > \min AC$ . These firms will make a slight profit.

**7.4.2 Final Comments on Partial Equilibrium**

The approach to partial equilibrium we have adopted has been based on a quasilinear model. How crucial is this for the results? Well, the quasilinear utility is not critical for the determination of

the competitive equilibrium – you would still find it in the same way. However, it is critical for the welfare results. Without quasilinear utility, the area under the Walrasian demand curves doesn't mean anything – so we will need a different welfare measure. Further, with wealth effects, welfare will depend on the distribution of the numeraire, not just the consumption good, which means that we will have to do additional work.