Hicksian Demand: Dependence on p and u.

Part 0: Recall that $\frac{\partial h_i}{\partial p_i} \leq 0$ because the Slutsky matrix is negative semi-definite. We'll assume, as is typical, that $\frac{\partial h_i}{\partial p_i} < 0$. To keep things simple, I'll omit the subscripts for the rest of the document, since we're always talk about a single good.

Part 1: Relationship between Hicksian and Walrasian Demand.

By duality, we know that through any point on Walrasian demand there is a Hicksian demand curve through that point. How do their slopes compare? This is given by the Slutsky equation. But, keep in mind two things. First, since we put p on the vertical axis and x on the horizontal axis, when we draw a graph, the derivatives of the demand functions aren't slopes. They're inverse slopes. That is, the slope of the Walrasian demand is $\frac{1}{\partial x/\partial p}$ and the slope of the Hicksian is $\frac{1}{\partial h/\partial p}$. Second, these derivatives are usually negative. So, we have to be a bit careful about thinking about quantities that are larger (i.e., further to the right on the number line) and quantities that are larger in magnitude (i.e., are further from zero on the number line). Since slopes are negative, a "larger" slope corresponds to a flatter curve. You'lls ee why this is important in a minute.

To figure out whether $x(p, w)$ or $h(p, u)$ through a point is steeper, use the Slutsky equation.

$$\frac{\partial h}{\partial p} = \frac{\partial x}{\partial p} + \frac{\partial x}{\partial w} x.$$

The answer will depend on whether x is normal or inferior. So, begin by considering a normal good. In this case, $\frac{\partial x}{\partial p} x > 0$, so:

$$\begin{array}{rcl} \frac{\partial h}{\partial p} & > & \frac{\partial x}{\partial p} \\ \left|\frac{\partial h}{\partial p}\right| & < & \left|\frac{\partial x}{\partial p}\right| \\ \frac{1}{\left|\frac{\partial h}{\partial p}\right|} & > & \frac{1}{\left|\frac{\partial x}{\partial p}\right|} \end{array}$$

The first line comes from the Slutsky equation and the fact that the income effect is positive. The second comes from the fact that, for a normal good, both sides are negative, and hence if $\frac{\partial h}{\partial p} > \frac{\partial x}{\partial p}$, $\frac{\partial h}{\partial p}$ is smaller in magnitude (absolute value) than $\frac{\partial x}{\partial p}$. The third follows from the second sinxe if $x < y$ and both are positve, then $1/x > 1/y$. Hence, for normal goods, the Hicksian Demand through a point is steeper than the Walrasian Demand through that point.

For an inferior good, things reverse. To simplify, suppose x is inferior but not Giffen (so that $\frac{\partial x}{\partial p} < 0$ – you can do the Giffen case on your own). In this case $\frac{\partial x}{\partial p} x < 0$, so:

$$\begin{array}{ccc} \dfrac{\partial h}{\partial p} & < & \dfrac{\partial x}{\partial p} \\ \left|\dfrac{\partial h}{\partial p}\right| & > & \left|\dfrac{\partial x}{\partial p}\right| \\ \dfrac{1}{\left|\frac{\partial h}{\partial p}\right|} & < & \dfrac{1}{\left|\frac{\partial x}{\partial p}\right|} \end{array}$$

and so Hicksian demand is flatter than Walrasian Demand.

Part 2: Dependence of Hicksian demand on u.

How does changing u shift the Hicksian Demand curve? Again, the answer depends on whether the good is normal or inferior. To see how, use duality:

$$h\left(p,u\right) \equiv x\left(p, e\left(p,u\right)\right),$$

and differentiate both sides with respect to u :

$$\frac{\partial h}{\partial u} \equiv \frac{\partial x}{\partial w}\frac{\partial e}{\partial u}.$$

By the properties of the expenditure function, we know that $\frac{\partial e}{\partial u} > 0$ (see MWG Prop 3.E.2, p. 59), so that $\frac{\partial h}{\partial u}$ has the same sign as $\frac{\partial x}{\partial w}$. Hence, when the good is normal, increasing u increases Hicksian demand **for any price.** Thus, increasing u shifts the Hicksian demand curve to the right. Similarly, when the good is inferior, increasing u decreases Hicksian demand for any price, and thus increasing u shifts the Hicksian demand for an inferior good to the left. The intuition is that in order to achieve a higher utility level, the consumer must spend more, and consumption increases with expenditure for a normal good and decreases with expenditure for an inferior good.

A couple of pictures. These pictures depict Walrasian and Hicksian demand before and after a price decrease for a normal good and for an inferior good. Note that $p^1 < p^0$ so that $u^1 > u^0$. For the normal good, Hicksian demand is steeper than Walrasian, and shifts to the right when the price decreases. For the inferior good, Hicksian demand is flatter than Walrasian and shifts to the left when the price decreases.

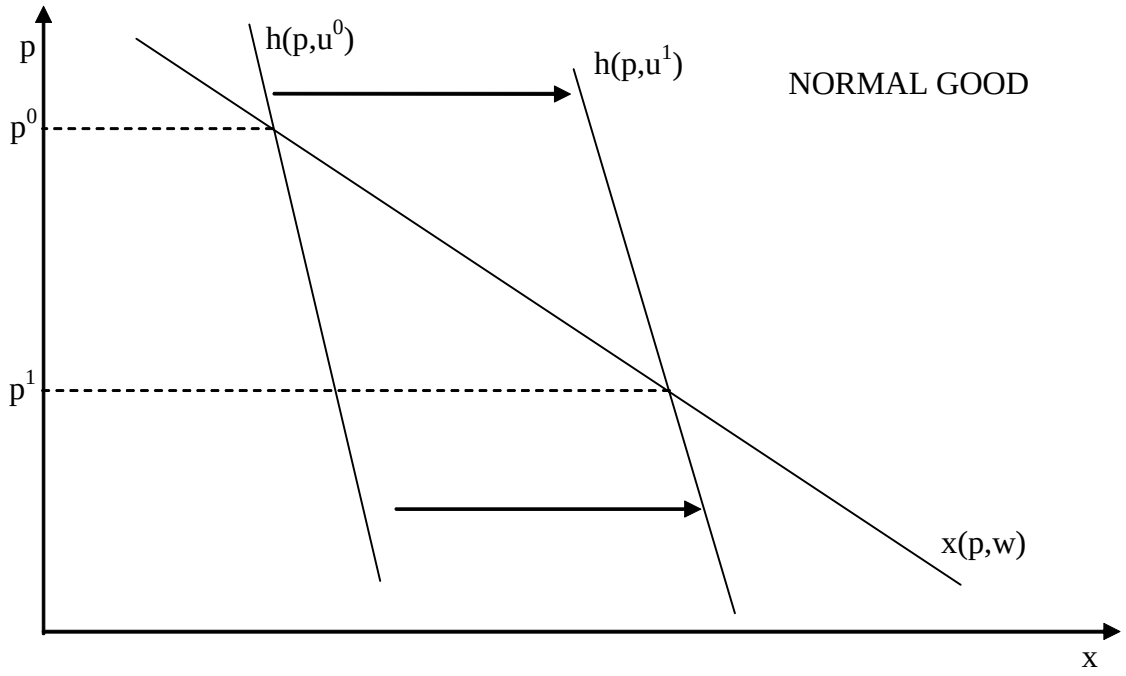

Figure 1:

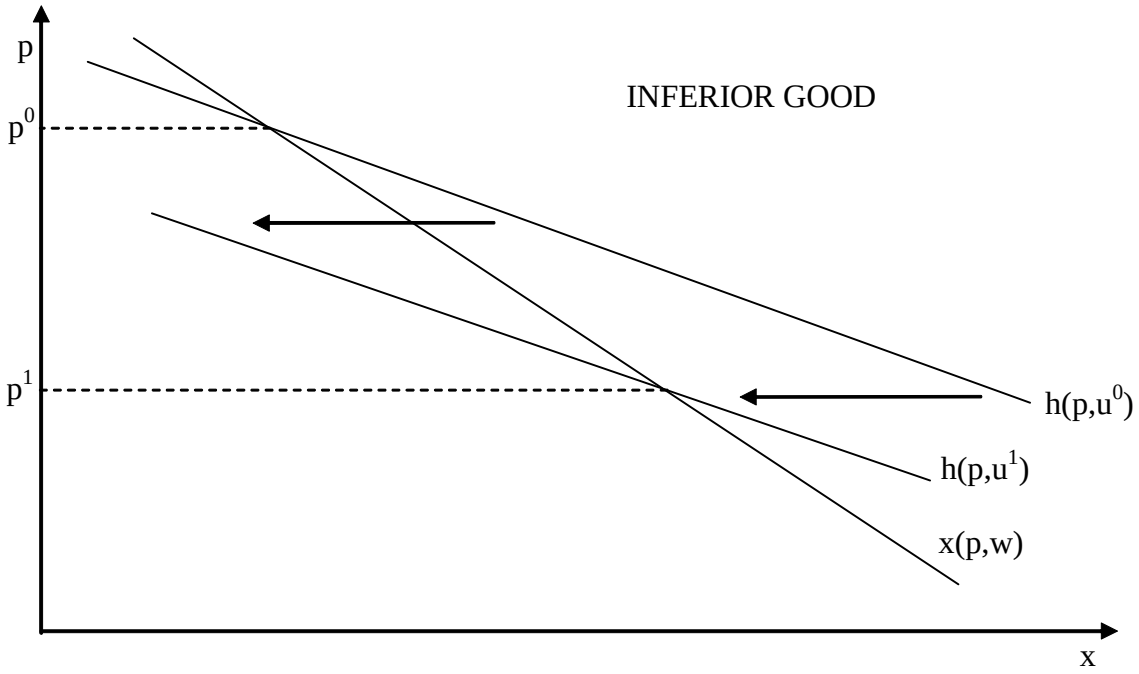

Figure 2: